**Forecasting Retailer Product Sales at the SKU level with the impact by the unobserved factors**

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Abstract

Retailers need accurate sales forecasts for their inventory management. In this study, we propose more effective methods to generate more accurate forecasts by taking into account the unobserved change in the effectiveness of the price and promotional activities. We implement the intercept correction method and the estimation window combining method to mitigate the issue of structural breaks and forecast bias. We evaluate our models for retailer products at the SKU level and we found our proposed new models with intercept corrections with the best forecasting performance.

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Key words:

Sales Forecasting, Marketing analytics, Promotion

1. **Introduction**

Retailers have been struggling with the situation of out-of-stocks and overstocks for years. When the product is out-of-stock, retailers not only immediately lose profits but also may lose the customers in the long term, as customers may purchase alternative products or postpone their purchases. Studies even show that customers may also switch to other stores and never come back ([Corsten and Gruen 2003](#_ENREF_19)). In practice, retailers may deliberately increase their inventory level (i.e. to over-stock) to avoid the out-of-stock situation. However, this significantly raises inventory costs and reduces profits ([Cooper, Baron et al. 1999](#_ENREF_17)). It is estimated that retailers in North American lost $634.1 billion due to out-of-stocks and $471.9 billion due to overstocks just in the year of 2014 ([OrderDynamics 2015](#_ENREF_59)). Under such circumstance, retailers need to balance the loss due to running out-of-stock and the cost of higher inventory level. One of the keys to resolve this cost and service level dilemma is to generate accurate forecasts for the product sales at the SKU level.

In practice, many retailers forecast product sales using a two-stage ‘base-lift’ approach at the SKU level. Specifically, they produce the baseline forecast using simple exponential smoothing methods and then make adjustments to the baseline forecast for any promotional event over the forecast period. The adjustments are usually made by brand/category managers with their experience. In the forecasting literature, a stream of studies has been devoted to helping managers improve their adjustment procedure (e.g., [Goodwin 2002](#_ENREF_29), [Fildes, Nikolopoulos et al. 2008](#_ENREF_25), [Nikolopoulos 2010](#_ENREF_58)). [Cooper, Baron et al. (1999)](#_ENREF_17) suggested estimate the adjustment with the model-based method and developed the PromoCast® forecasting system. Other studies proposed holistic sophisticated models (e.g., neural networks and machine learning algorithms) which directly generate the final forecasts for product sales (e.g., [Aburto and Weber 2007](#_ENREF_1), [Gür Ali, SayIn et al. 2009](#_ENREF_30)). [Huang et al. (2014)](#_ENREF_34) proposed general-to-specific Autoregressive Distributed Lag models which incorporate the promotional information of not only the focal product but also of the competitive products within the same product category. ([Ma, Fildes et al. 2016](#_ENREF_47)) further integrated the promotional information from the products across different categories.

All the studies mentioned above assume invariant effectiveness of the promotional activities. In practice, this may not be true because of the impact of many influencing factors including the change of economic conditions, the change of consumer tastes, and media habits, new competitor entry etc. ([Wildt 1976](#_ENREF_71), [Wildt and Winer 1983](#_ENREF_72)). The business environment changes quickly and magnificently. A recent example is that the low-price German retailer Aldi which had huge expansions across Europe has opened more than 400 stores in the United States just in the year of 2014 ([Loeb 2015](#_ENREF_46)). Under such circumstance, conventional models may potentially be subject to structural break which is defined as large change in the model with respect to the parameter coefficients ([Armstrong 2001](#_ENREF_8)). As a result, the model may produce biased and less accurate forecasts. The issue of structural break and the opportunity to improve forecasting performance by mitigating the consequent forecast bias have been intensively addressed in the macroeconomics literature ([see Clements and Hendry 1994](#_ENREF_14)). However, this issue has been totally overlooked in the area of forecasting retailer product sales.

In this study, we aim to improve the forecasting performance for retailer product sales at the SKU level by taking into account the unobserved change of the effectiveness of the price and promotional activities. We propose models which generate more accurate forecasts by mitigating forecast bias caused by potential structural breaks using two methods. For the Intercept Correction (IC) method, we make adjustments to the out-of-sample forecasts based on an estimate of forecast bias. For the estimation window combining (EWC) method, we take a trade-off between the forecast bias and the forecast error variance by combing the forecasts generated by the same model but with different estimation windows. These two methods have been successfully applied with VAR models in forecasting macroeconomic data. In the retailer’s context, product sales at the disaggregate SKU level contain huge variations which are very different from macroeconomic data in terms of data characteristics. As a result, the potential forecast bias may be submerged in the variation of the sales, and the potential improvement by using the two methods may to be small compared to the variation of the product sales. Therefore, whether we can generate more accurate forecasts for retailer product sales by taking into account the issue of structural break with these methods is an empirical question. Methodologically our study conducts a novel evaluation of the method which leads the combine of dynamic models and variable selection process a dimension forward by taking into account the unobserved change of the effectiveness of the explanatory variables.

We implement the EWC method and the IC method based on Autoregressive Distributed Lag (ADL) models reduced by the LASSO selection procedure following Ma et al. (2016). The results indicate that the ADL model with the EWC method generate the most accurate forecasts, and the IC method improve. Overall, the ADL-IC model generate the most accurate forecasts for retailer product sales at the SKU level. We also see that the ADL-own-IC model with the best forecasting performance when competitive information is not an option. Therefore, our models have practical significance in that they equip retailers as well as manufacturers the tools to forecast product sales and more effectively manage their inventory planning.

The remainder of the paper is arranged as follows: section 2 summarizes previous research findings. Section 3 introduces the issue of structural break and resulted forecast bias when conventional models overlook the change in the effectiveness of the price and promotional activities. In section 4, we introduce two methods from the macroeconomic literature to mitigate the issue of structural break. Section 5 and section 6 describe the data and the candidate models respectively. Section 7 introduces the experimental design for the evaluation. In section 8, we summarize and discuss the evaluation results. In section 9, we draw conclusions and make recommendations for both manufacturers and retailers, and we also address some research limitations and possibilities for future research.

**2. Literature review**

2.1 Forecasting retailer product sales at the SKU level

In practice, many retailers produce forecasts for retailer product sales at the SKU level using the ‘base-lift’ approach. For example, they first generate the ‘baseline’ forecasts using univariate methods but excluding the data when the focal product is being promoted. They then make adjustments to the baseline forecast if there is an incoming promotional event in the future ([Fildes, Nikolopoulos et al. 2008](#_ENREF_25), [Fildes, Goodwin et al. 2009](#_ENREF_24)). The univariate models for the ‘baseline’ forecast are usually simple such as the simple exponential smoothing method, though evidence suggests that these simple models can be hard to defeat when the sales data are relatively stable (e.g., over the forecast period when the focal product is not being promoted) ([Gür Ali, SayIn et al. 2009](#_ENREF_30)). The adjustments to the incoming promotional event, which are usually done by brand/category managers, are prone to systematic bias and associated with high costs ([Fildes, Goodwin et al. 2009](#_ENREF_24)). A stream of studies has been devoted to helping managers with their judgmental procedure ([Arenas, Pedregal et al. 2013](#_ENREF_7)). Some other studies try to improve the adjustment with model-based forecasting systems which estimate the ‘lift’ effect by the promotional event using historical information related to previous promotions, store/category features, and manufacturers ([Cooper, Baron et al. 1999](#_ENREF_17), [Cooper and Giuffrida 2000](#_ENREF_18), [Trusov, Bodapati et al. 2006](#_ENREF_66)). However, there is an intrinsic limitation for these methods of two stages (i.e., baseline with adjustments) because they produce forecasts separately depending on whether the focal product is being promoted or not. Under such circumstance, the information when the focal product is being promoted are inevitably overlooked when forecasting the sales of the product when the product is not being promoted, and vice versa.

Some studies proposed holistic methods which relies on sophisticated data mining and machine learning algorithms. [Aburto and Weber (2007)](#_ENREF_1) evaluated the performance of neural network algorithms in forecasting supermarket food sales. [Gür Ali, SayIn et al. (2009)](#_ENREF_30) proposed support vector machine models and the regression tree models to forecast retailer product sales at the SKU level. They also constructed a range of variables based on the promotional information of the focal product (i.e. average, sum, trend, standard deviation of previous sales and price etc.). Other studies proposed econometric forecasting models which not only benefit from high forecasting accuracy but also interpretability which is critically important for practical application. Divakar et al. (2005) proposed the CHAN4CAST system with dynamic regression models to forecast brand sales for manufacturers/channels. More recent studies tried to increase forecasting accuracy through the direction of incorporating more information. For example, [Huang, Fildes et al. (2014)](#_ENREF_34) included competitive promotional information within the same product category to forecast the sales of the focal product. They dealt with the high dimensionality problem specific to the retailer context at the SKU level with variable selection methods and principle component analysis. [Ma, Fildes et al. (2016)](#_ENREF_47) further integrated the promotional information not only from the same product category of the focal product but also from other related categories. They rely on Granger causality test to find out the relevant product categories and then adopted the LASSO algorithm not only for variable selection but also as a model specification strategy.

2.2 The changing effectiveness of marketing activities

Many studies in the marketing literature have explored the effect of promotional activities. For example, early studies found that promotions significantly increase short-term sales of the focal product (Blattberg, 1995). Evidence also show that promotions have positive (negative) impact on complementary (competitive) products ([Wittink, Addona et al. 1988](#_ENREF_74), [Dekimpe, Hanssens et al. 1999](#_ENREF_21), [Andrews, Currim et al. 2008](#_ENREF_6)). The impact of promotions can be asymmetrical as promotions on national brands have much stronger effect on store-label brands (Wedel and Zhang 2004). Promotions also have dynamic effects. For example, promotions may either accelerate customers’ consumption or postpone their purchases if customers anticipate future promotional events ([Van Heerde, Gupta et al. 2003](#_ENREF_67), [Mace and Neslin 2004](#_ENREF_48)).

Some other studies focused on investigating the change in the effectiveness of the promotional activities (e.g. [Little 1966](#_ENREF_45), [Morrison 1966](#_ENREF_53), [Myers and Nicosia 1970](#_ENREF_56), [Myers 1971](#_ENREF_55), [Houston and Weiss 1975](#_ENREF_33), [Monroe and Guiltinan 1975](#_ENREF_52), [Moinpour, McCullough et al. 1976](#_ENREF_51), [Wildt 1976](#_ENREF_71), [Wichern and Jones 1977](#_ENREF_70), [Winer 1979](#_ENREF_73), [Mahajan, Bretschneider et al. 1980](#_ENREF_49)). They argue that the effectiveness of promotions may change because of various reasons. For example, the change in economic condition, legislation, consumer tastes, media habits, and advertising etc. ([Wildt 1976](#_ENREF_71), [Wildt and Winer 1983](#_ENREF_72)). Also, the effectiveness of promotions may change with the different stages of the product life cycle ([Mahajan, Bretschneider et al. 1980](#_ENREF_49)). Marketing theory suggests that the elasticities for marketing instruments (e.g. advertising, price, service, product quality, and packaging) are the highest at the growth stage of the product and the lowest at the maturity stage of the product ([Kotler 1997](#_ENREF_40)). The impact by other influencing factors such as new competition may also change the effectiveness of some marketing activities. e.g., the introduction of new products (especially the store-owned brand) decrease promotional elasticities of premium national brands and increase promotional elasticities of the second tier national brands ([Nijs, Dekimpe et al. 2001](#_ENREF_57), [Van Heerde, Srinivasan et al. 2008](#_ENREF_68)). Further evidence indicates that intensive promotions can make consumers less responsive to price promotions by reducing consumers’ reference price ([Lattin and Bucklin 1989](#_ENREF_41), [Lichtenstein and Bearden 1989](#_ENREF_44), [Kalwani, Yim et al. 1990](#_ENREF_37), [Kalwani and Yim 1992](#_ENREF_36), [Foekens, Leeflang et al. 1999](#_ENREF_28), [Kopalle, Mela et al. 1999](#_ENREF_39), [Levy, Grewal et al. 2004](#_ENREF_43)). The introduction of a new distribution channel (e.g., online website) can also reduce the effectiveness of price promotions on the original channel ([Verhoef, Neslin et al. 2007](#_ENREF_69)). This is because that consumers may get more easily to collect information through the newly constructed channel and then reduce their reference price. Consumers’ response to the price reductions and promotions by competitive products may be changed by the introduction of loyalty programme of the focal product ([Leenheer, van Heerde et al. 2007](#_ENREF_42)). When consumers become loyalty programme members of a specific brand, they receive saving rewards and direct discounts, and may find the promotions of alternative brands less attractive. This also applies to the termination of existing loyalty program ([Melnyk and Bijmolt 2007](#_ENREF_50)).

Some studies tried to capture the changing process of the effectiveness of the marekting actitives. Foekens, S.H. Leeflang et al. ([1999](#_ENREF_28)) extended the original SCAN\*PRO model to incorporate the time-varying effects of the marketing mix variables. In the extended model, the parameters of the marketing mix variables are modelled as a function of historical promotional information of the focal brand and other competitive brands. The model tries to capture how the effects of the marketing mix variables change over time so that managers can allocate the marketing budget more efficiently. Kopalle, Mela et al. ([1999](#_ENREF_38)) also proposed extensions of the SCAN\*PRO model to investigate the dynamic impact of promotions on the baseline sales. Their results indicate that promotions increase the concurrent product sales but reduce the baseline sales. These models, however, are all descriptive models and are not used in forecasting retailer product sales.

1. **Promotions, structural break, and forecast bias**

What is SB

When the effectiveness of the price and promotions on product sales change, as described in previous section, conventional forecasting models will be subject to structural break which is defined as large changes in the model’s parameters ([Allen and Fildes 2001](#_ENREF_3)). The parameter estimates of the models then become the weighted average of the true parameters before and after the structural break. The forecasts generated by the model will be subject to bias and less accurate[[1]](#footnote-1). The impact of structural break on the model’s forecasting performance has been addressed by many studies in the macroeconomics literature (e.g. [Cooper and Nelson 1975](#_ENREF_16), [Muellbauer 1994](#_ENREF_54), [Hendry 1995](#_ENREF_31), [Clements and Hendry 1999](#_ENREF_15), [Pesaran and Timmermann 2007](#_ENREF_60), [Castle, Doornik et al. 2008](#_ENREF_11)).

[Pesaran and Timmermann (2005)](#_ENREF_62) provided the analytical evidence for the impact of one single structural break on the forecasting performance of a simple regression model. In our context, suppose we have sales and price data for an SKU from week 1 to week *T,* i.e., and a structural break occurs at week (where ). We also assume that the parameter of the price variable changes from to after . In practice, this may be caused by the impact of a new brand entry, a new advertisement by other existing brands, and even the change of the temperature (especially for product categories such as soft drinks and ice creams) etc. The information may not be observable to us. We may assume the following real demand:

where, is an indicator which equals to 1 before week and 0 otherwise. and are respectively the sales and the price of the product at week *t*. We assume that retailers do not change product price based on their short-term sales, thus is considered as strictly exogenous[[2]](#footnote-2). and are the parameters before and after the structural break at week . is the error term, and when and when .

We may estimate a model with a functional form congruent with the demand (e.g., ) where the estimation window starts before the structural break, e.g., at week *m* . The OLS estimate for the model is therefore:

where and are the matrices for the sales and price respectively with the observations from week *m* to week T. Under such a circumstance, is a weighted average of and . We assume no structural break after week T, and the true demand after week T will remain as . Therefore, the *h*-step ahead forecast error at week *T*+h (with *m* as the starting observation of the estimation window) can be represented as:

where , is the matrix for the error term at week .

Accordingly, the forecast bias at week can be represented as , and is unequal to zero as .

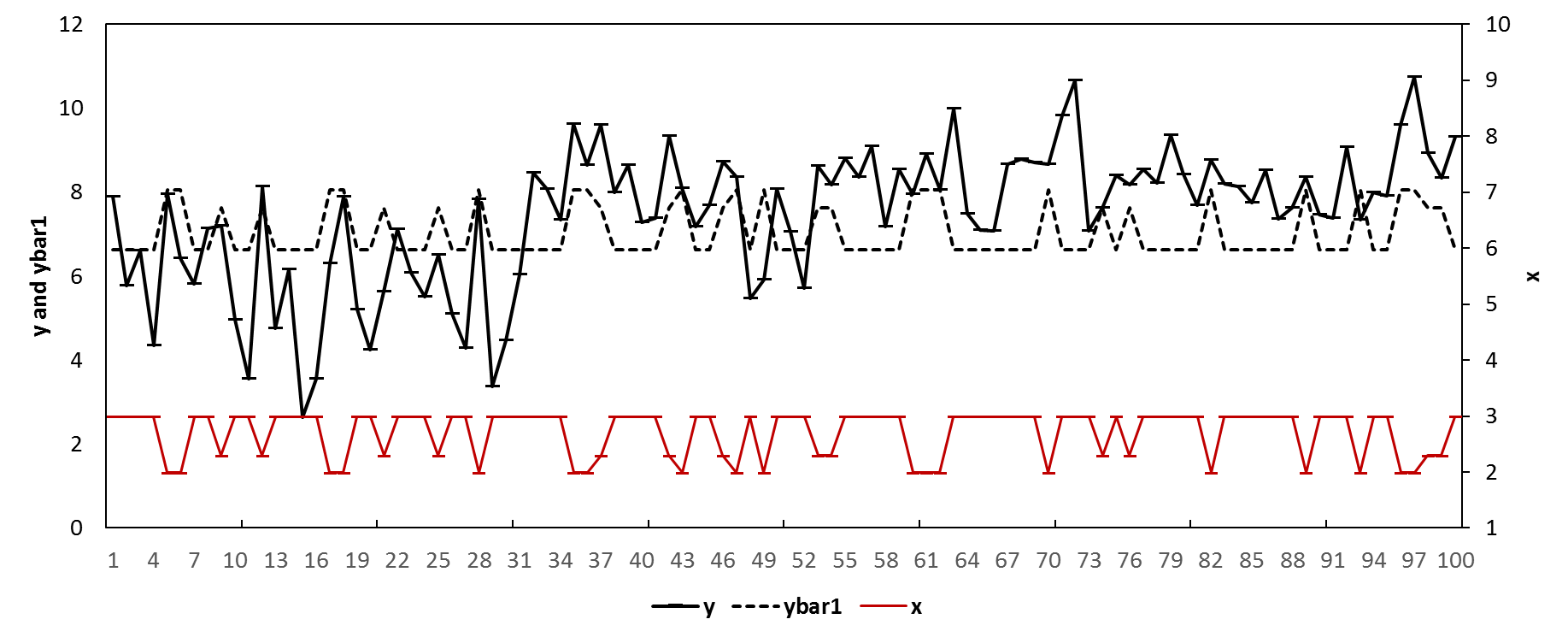
The forecast bias and the subsequent inferior forecasting performance can be further illustrated using an example based on simulation. We generate artificial price which is 2.99 for most of the observations but occasionally reduced to 2.29 or 1.99[[3]](#footnote-3). i.e., or 2.29 or 1.99. We assume the product sales to be exclusively determined by the price but with a structural break at week 31. This can be represented as follows:

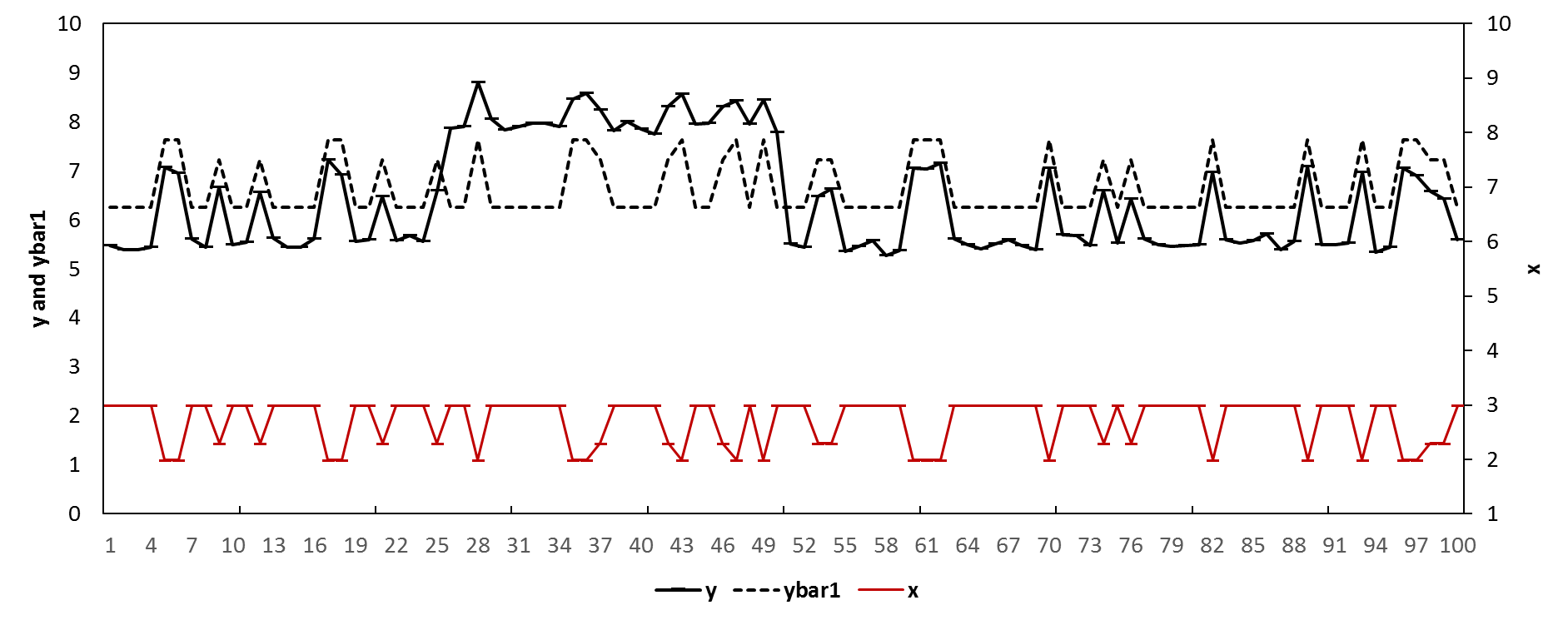
, , when

, , when

Based on the simulated data the product sales increase but also become less responsive to temporary price reductions after the break. This is reasonable especially for those products in the growth to mature stage in their product life cycle[[4]](#footnote-4). The sales and price data are represented in Figure 1 by the solid black line and the solid red line respectively.

Figure 1. Simulated sales with a structural break: model with full data

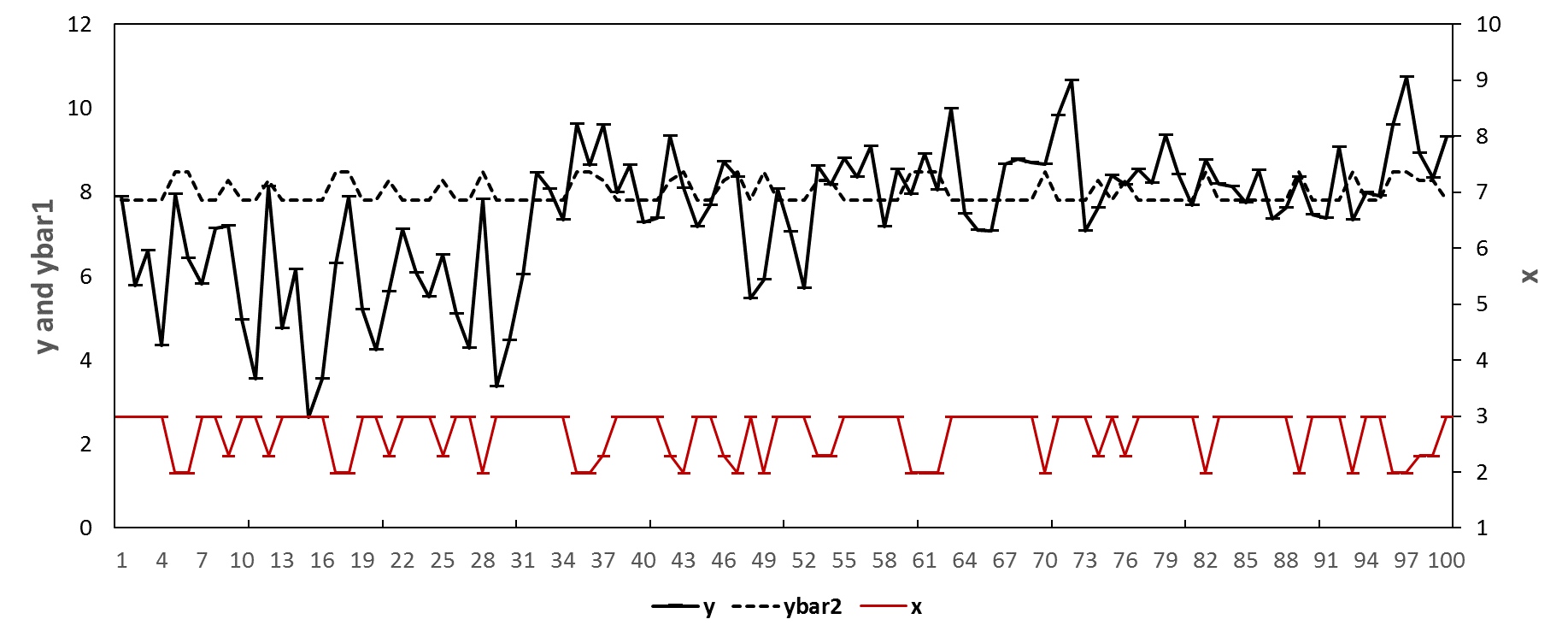




Suppose we have the data from week 1 to week 70 and we want to forecast the product sales from week 71 to week 100. Therefore, in Figure 1, the blue area represents the time period before the structural break, the yellow area represents the estimation period after the structural break, and the red area represents the forecast period. We may estimate the model with the function form as using the data from week 1 to week 70 but overlook the change of the real effect of the price at week 31. Under such circumstance, we will have estimates as the weighted average of the true parameters before and after week 31. As a result, we will over-predict the product sales for the time period from week 1 to week 30 and under-predict the product sales for the time period from week 31 to week 70, and we will produce downwards-biased forecasts for the time period from week 71 to week 100. The predictions/forecasts are represented by the black dashed line (as *ybar1*) in Figure 1. Table 1 shows the forecasting performance of this model regarding some error measures[[5]](#footnote-5).

Alternatively, we may estimate the model exclusively using the data from week 31 to week 70 and then generate forecasts which are unbiased. These forecasts are represented by the black dashed (as *ybar2*) line in Figure 2. However, in a retailing context. There are so many factors which may change the effect of the price, as mentioned early in section 2. Therefore, the structural break at week 31 is not observable. Also, if the structural break occurs at an observation which is close to the forecast origin, there will not be enough post-break observations to estimate the model.

Figure 3. Simulated sales with a structural break: model with post-break data



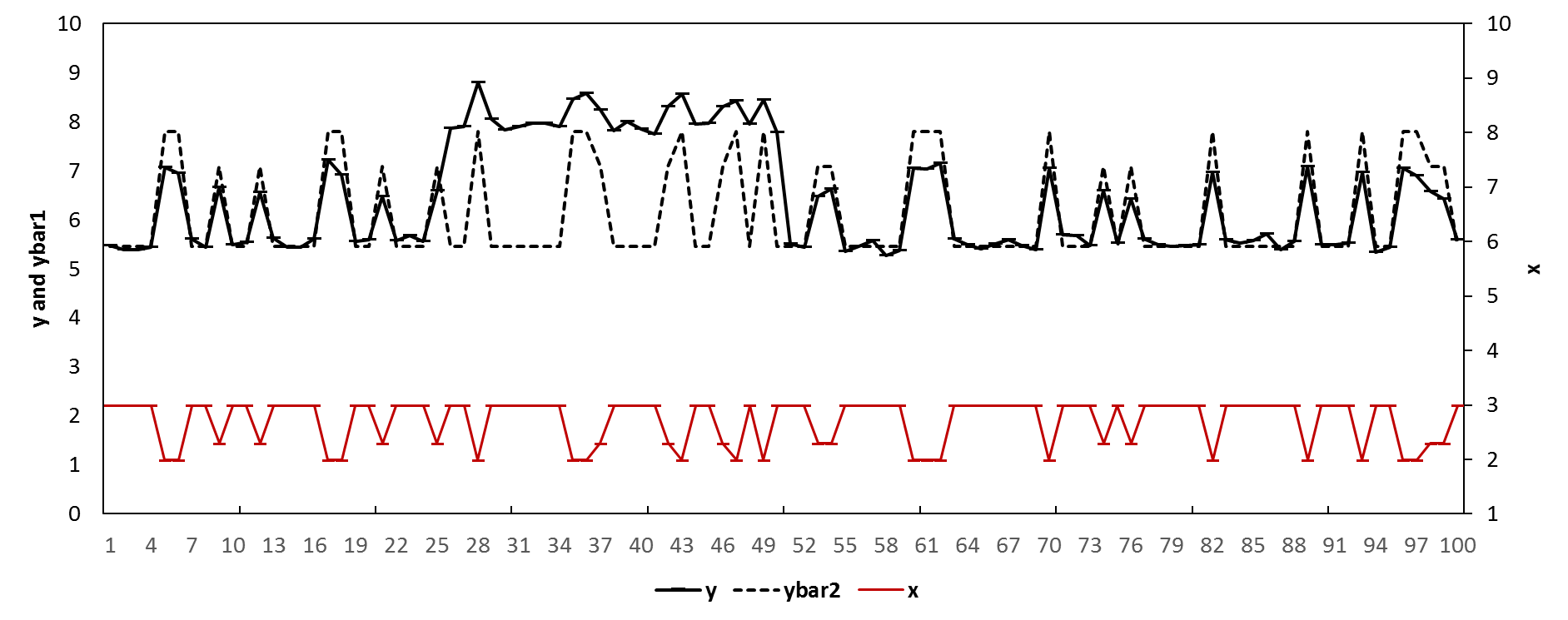


Table 1. The forecasting performance of different models in the simulation

|  |  |  |  |
| --- | --- | --- | --- |
| Model | MAE | MAPE | SMAPE |
| Figure 1: Model with full estimation window | 0.715 | 12.2% | 11.5% |
| Figure 2: Model with Post-break estimation window | 0.294 | 5.0% | 4.3% |
| Figure 3: Model with intercept correction | 0.101 | 1.7% | 1.8% |
| Figure 4: Model with estimation window combining | 0.647 | 11.0% | 10.5% |

1. **The methods**

4.1 Intercept correction

In section 3, the model which overlooks the structural break generates biased and less accurate forecasts. Previous studies which forecast macroeconomic data series tended to estimate the forecast bias and then offset the forecast bias (e.g., regime shifts) by specifying non-zero values for the model’s errors in the forecasting period ([Clements and Hendry 1994](#_ENREF_14), [Clements and Hendry 1999](#_ENREF_15), [Clark and McCracken 2007](#_ENREF_12)). This method is referred as intercept correction (IC) method which may potentially improve the forecasting accuracy by mitigating the forecast bias but at the cost of inflated forecasting error variance ([Clements and Hendry 1999](#_ENREF_15)).

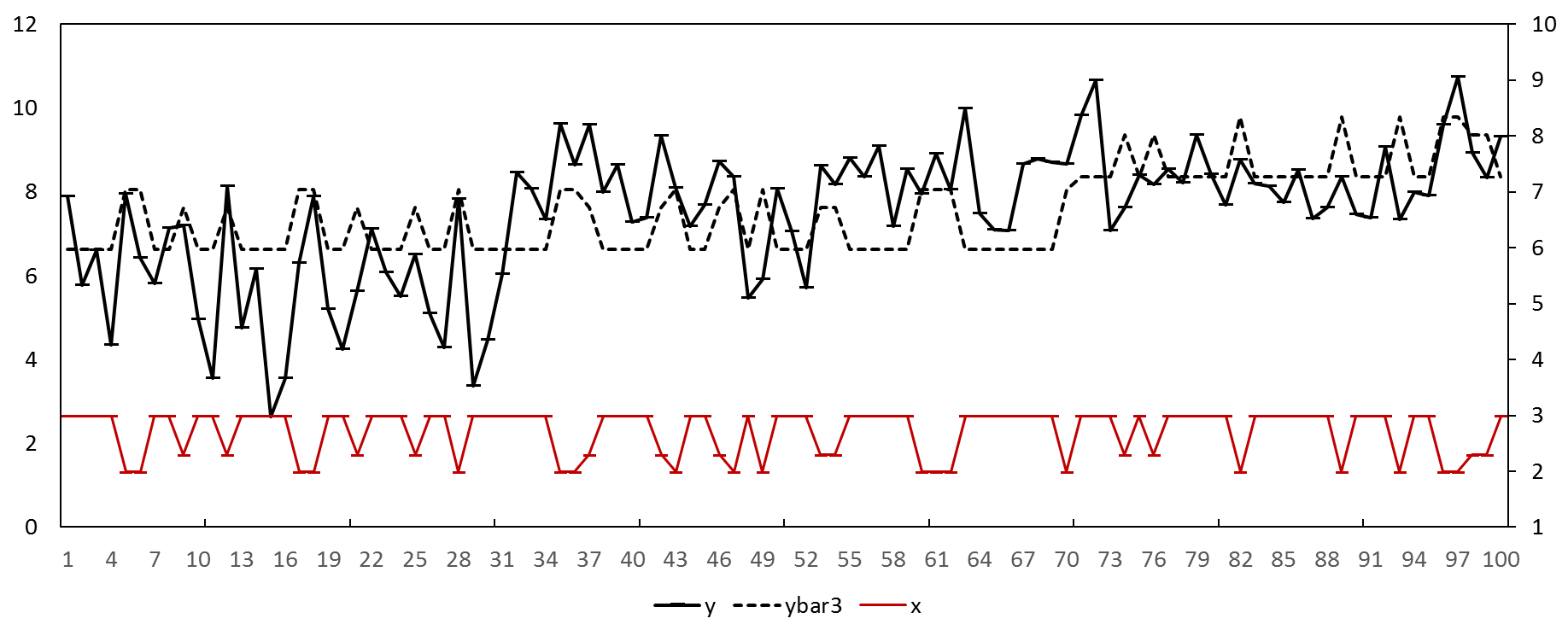
The IC method can be demonstrated using the same example described in section 3 where we have the model as but no prior knowledge related to the existence nor the location of the structural break. We first conduct a sequential Chow (1960) test based on most of the observations of the estimation period[[6]](#footnote-6). Figure 3 shows the *p*-values of the sequential Chow test assuming there is one single structural break occurring at a specific week. The results reject the null hypothesis of no structural break for the estimation period. The sequential Chow test does not suggest the location of the structural break but indicate whether or not a structural break may exist during the estimation period[[7]](#footnote-7). In the literature, more advanced statistic tests have been proposed to detect the locations of the structural breaks but they all need to assume additional priori knowledge such as the number of potential structural breaks ([Andrews 1993](#_ENREF_4), [Andrews and Ploberger 1994](#_ENREF_5), [Bai and Perron 2003](#_ENREF_9)).

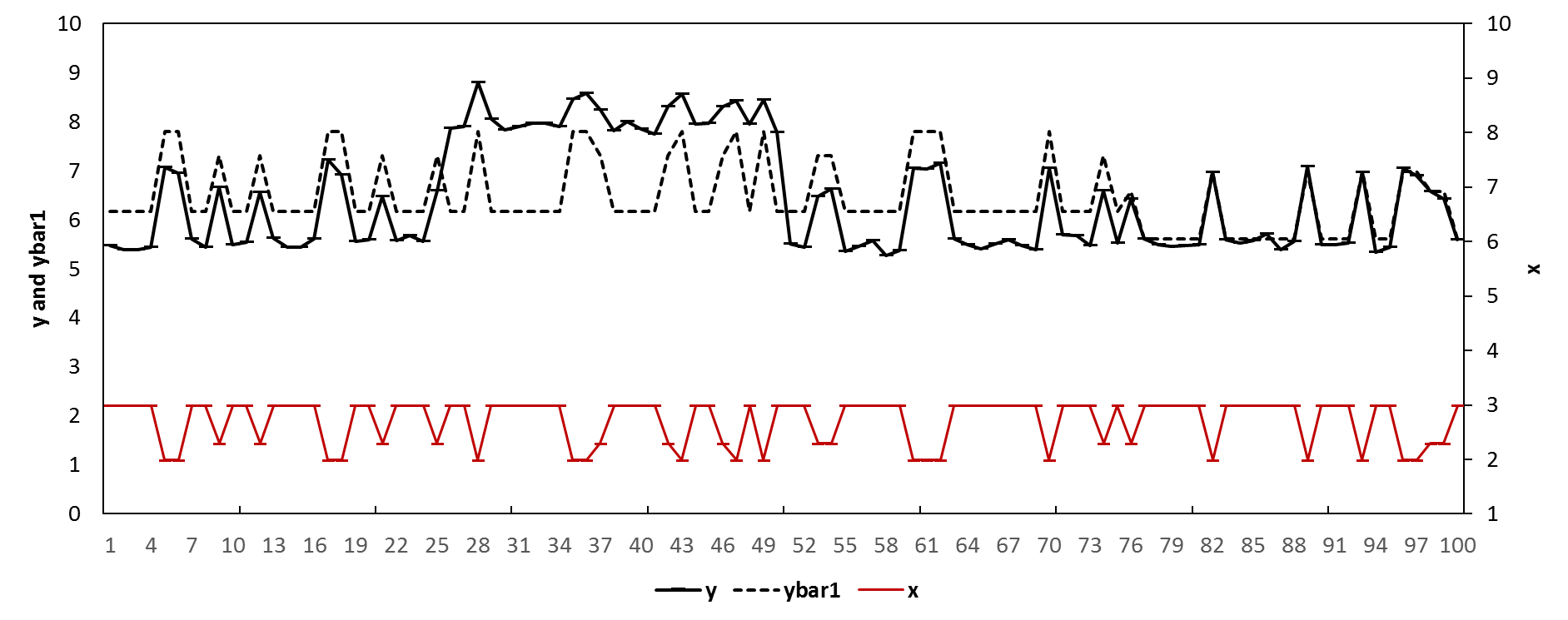
Figure 4 P-values of the sequential Chow test



Based on the results in Figure 3, we consider the model to be subject to structural break and generate biased forecasts. We may then estimate the forecast bias with different schemes. For example, we may estimate the bias as the predictive error at the forecast origin (i.e., , where *T* =70). Alternatively, we may estimate the forecast bias as the average value of an ad hoc number of predictive errors before the forecast origin. (e.g. , where *i* can be arbitrarily chosen). In this example, we estimate the forecast bias as the average of the predictive errors for the last four observations in the estimation period. e.g., .

Figure 5. Simulated sales with a structural break: model with intercept correction





We then add the bias estimate back to the forecasts. e.g., , where represents the final ‘intercept corrected’ forecasts which are represented by the black dashed (as *ybar3*) line in Figure 4. These forecasts are more accurate compared to those generated by the original model (e.g., with MAE= 0.824, MAPE= 9.77%, and SMAPE= 9.54%, as shown in Table 1).

One of the limitations for the IC method is that it heavily relies on the detection and the estimation of the forecasts bias. In the retailing context, the product sales at the SKU level have large variations and may immerge the forecast bias. Also, the IC method mitigate the forecast bias by adding the estimated bias back to the forecasts but at a cost of inflated error variance of the forecasts (Clements and Hendry ([1999](#_ENREF_15)). Whether we can generate more accurate forecasts by implementing the IC method to conventional models for retailer product sales at the SKU level is an empirical question.

4.2 Estimation window combining

The intercept correction method may potentially improve the forecasting performance of conventional models, but relies on how accurate the structural break can be detected and how accurate the forecast bias can be estimated. An alternative method which circumvents the estimation of the forecast bias is to combine the forecasts generated by models with the same specification but with different estimation windows. As described in section 3, if we know the location of the structural break, we could estimate the model exclusively with the post-break data (e.g., the data from week 31 to week 70), and the model will not be subject to structural break and will generate unbiased forecasts. In reality, we do not know whether the structural break exists nor the location of the potential structural break. We may estimate the model with the most recent observations close to the forecast origin. The model will be less likely to be subject to structural break. For the model in section 3, we may keep *m* as large as possible (so that we discard more old data) to generate less biased forecasts. When *m* becomes larger than , the model will be estimated exclusively with post-break data, and will generate unbiased forecasts. We may arbitrarily choose to estimate the model using the data from week 50 to week 70 even when we do not observe the date of the structural break.

However, the reduction of forecast bias comes with the cost of inflated forecasting error variance as we estimate the model using less information. In the same example in section 3, the forecast error is represented as follows:

The corresponding Mean Square Error (MSE) at week , is represented as:

where

is interpreted as the squared forecast bias, and is interpreted as the efficiency term ( is the forecasting error variance). The change of the MSE for week when we include one more observation in the model estimation is:

where is the MSE for the model which is estimated with the data from week m-1 to week T. [Pesaran and Timmermann (2007)](#_ENREF_60) show that the bias term () (i.e., the change of the squared forecast bias) is always larger than or equal to zero (i.e., with one more observation before the structural break, the forecast will get more biased), but the sign for the efficiency term depends on the percentage of the change in the error variance before and after the structural break (i.e., )). If (e.g., there are more post-break variations in the product sales which cannot be explained by the price variable), will be larger than or equal to , and the MSE may increase as both terms are non-negative. However, if (e.g., there are less variations in the product sales which cannot be explained by the price variable), may be smaller than or equal to . Under this condition, the MSE will either increase or decrease depending on how the non-negative squared bias term compares to the non-positive efficiency term. Therefore, when we include pre-break data in the model estimation, we may have either better or worse forecasting performance depending on the trade-off between the increased forecast bias and the potentially reduced forecasting error variance.

Under such circumstance, we may resort to forecasting combination [ref] which combine the forecasts generated by the models with different estimation windows ([Pesaran and Timmermann (2007)](#_ENREF_60). In this study, we implement the combination scheme with equal weights because it has been proved with good performance and easy to implement.([Clements and Hendry 1998](#_ENREF_13), [Fildes and Stekler 2002](#_ENREF_26), [Dekker, van Donselaar et al. 2004](#_ENREF_22), [Pesaran, Schuermann et al. 2009](#_ENREF_61)). Specifically, we estimate the model using the most recent observations (e.g., the estimation window contains the observations from week to week ) to generate the first set of *h*-step-ahead forecast as:

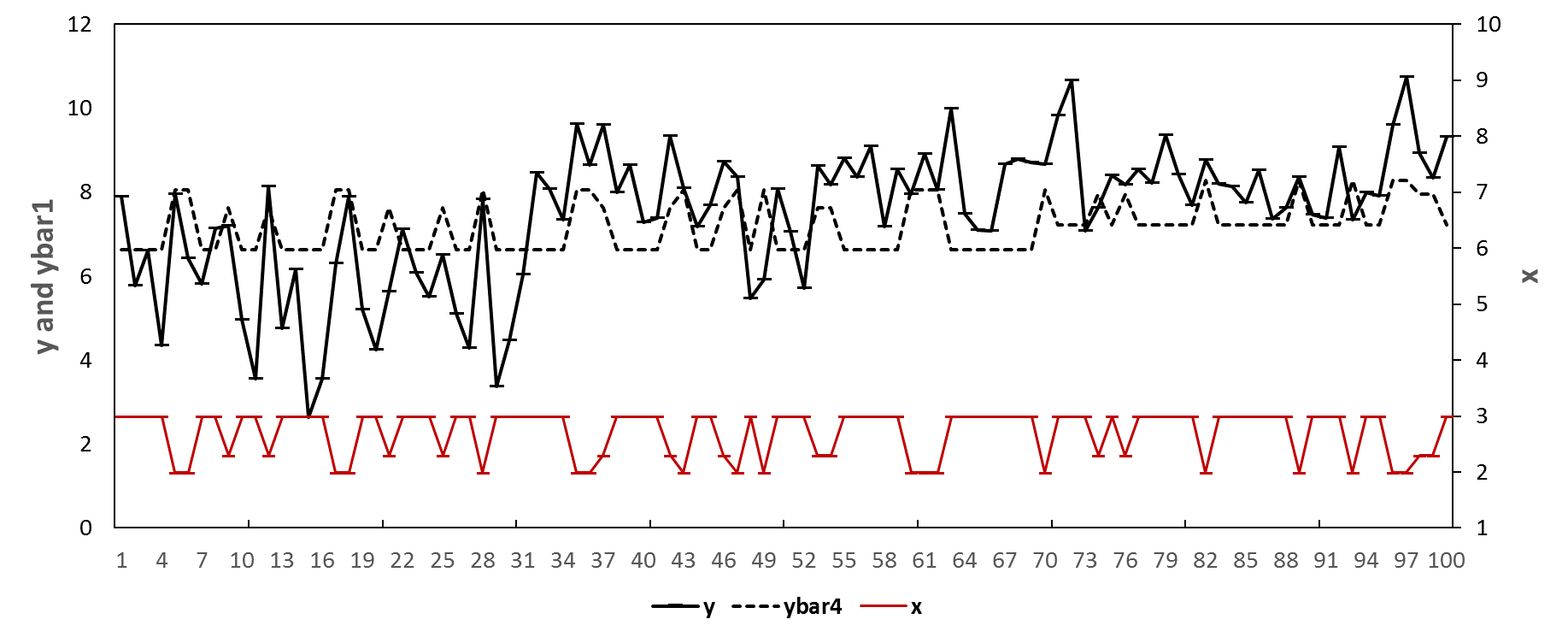
where the value of is arbitrarily chosen given there are enough observations to estimate the model and there are enough variations for all the explanatory variables. We then re-estimate the model and re-generate forecasts by adding more observations to the estimation window and. For example, we may have the set of *h*-step-ahead forecast as:

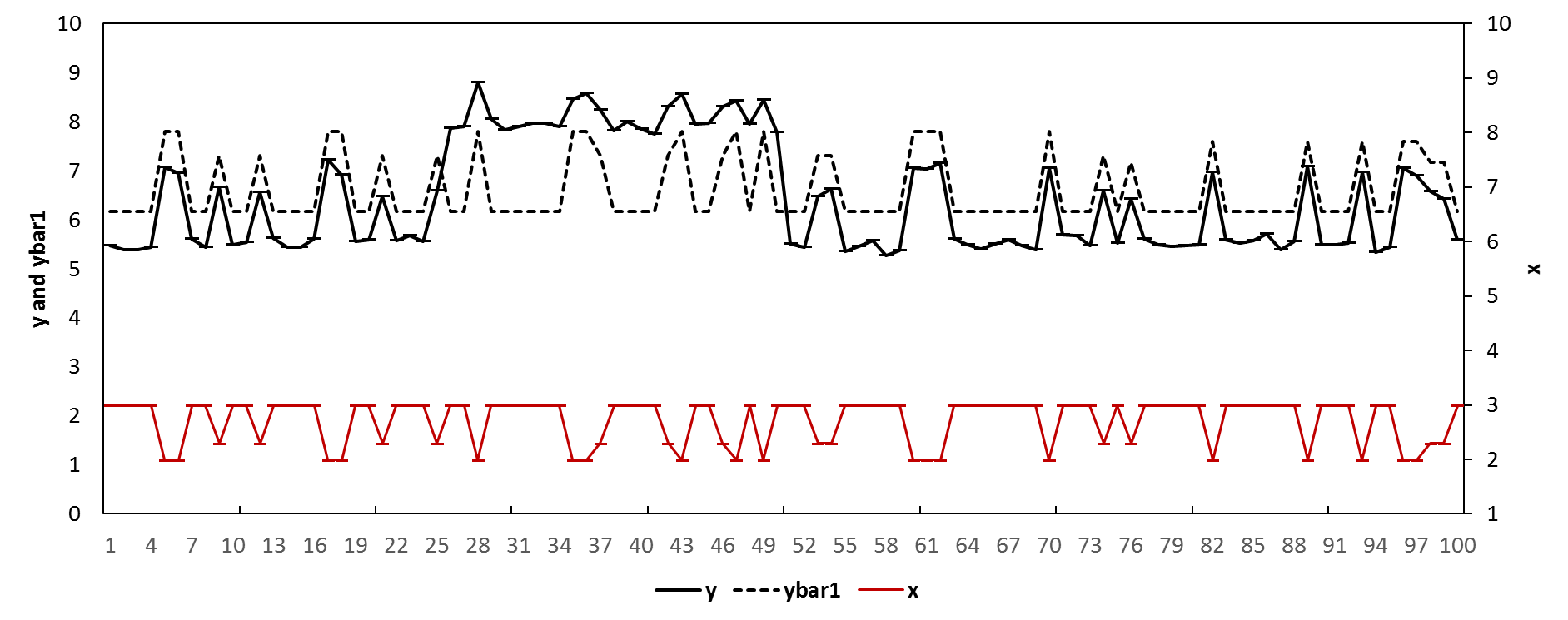
Eventually, we combine the () sets of *h*-step-ahead forecasts based on equal weights:

This can be illustrated with the same simulation example in section 3. Suppose that there is a structural break within the estimation period but we do not know the date of the break is at week 31. We may estimate the model with different lengths of estimation windows and combine their forecasts. For example, we first estimate the model using the data from week 1 to week 70, and generate the forecasts for the period after week 70. We denote this set of forecasts as which are subject to the full bias. We then estimate the same model but using the data from week 2 to week 70, and generate forecasts for the period after week 70 and denote them as , and so forth. The forecasts such as will be less biased compared to but associated with inflated forecasting error variance because they were generated by models with less information. can be arbitrarily chosen given there are enough observations and variations to estimate the model. In this simulation, we choose *n* to be 40 and we combine the 40 sets of forecasts with equal weights. i.e.,. where is the forecasts by the EWC method for week *t*. are illustrated by the black dashed line in Figure 5. The forecasts are more accurate compared to the forecasts by the original model shown in Table 1. (e.g., 1.034 for MAE, 12.17% for MAPE, and 12.58% for SMAPE).

The EWC method relies on the trade-off between the reduced forecast bias and the inflated forecast error variance. In this study, we evaluate the empirical question that whether we can generate more accurate forecasts by implementing the EWC method to conventional models for retailer product sales at the SKU level.

Figure 6. Simulated sales with a structural break: model with estimation window combing





1. **The data**

We evaluate our models using the retailing dataset by the IRI company. The dataset has been briefly introduced by [Bronnenberg, Kruger et al. (2008)](#_ENREF_10)[[8]](#footnote-8). It contains weekly data at the SKU level with variables including unit sales, price, features and displays etc. for more than seven years. We conduct our evaluation based on 1834 SKU’s with positive movements for at least 90% of time for 30 product categories from 30 stores. Table 2 shows the basic statistics for the selected SKU’s for each product category. The table indicates that some product categories (e.g., Carbonated beverages and Hotdog) have much higher promotional intensity compared to other categories (e.g., Margarine/Butter and Mayonnaise). Figure 6 depicts the sales data for a typical SKU in the Beer category. The product has occasional price reductions and feature/display events where the product sales exhibits spikes accordingly.

Table 2. The statistics for the SKUs in each of the product categories

Table 2. Statistical description for the product in the categories

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Category | Price mean | Price standard deviation | Price coefficient of variation | Sales mean | Sales standard deviation | Sales coefficient of variation | Display percentage | Feature percentage | Outliers percentage | Randomness | Linear trend | Number of SKU's |
| Beer | 8.34 | 0.51 | 0.06 | 20.61 | 13.05 | 0.61 | 13.9% | 4.0% | 4.4% | 0.19 | -0.05 | 169 |
| Blades | 8.13 | 0.40 | 0.06 | 14.59 | 6.38 | 0.53 | 7.4% | 2.2% | 2.9% | 0.21 | -0.01 | 22 |
| Carbonated Beverages | 2.10 | 0.32 | 0.14 | 113.59 | 153.42 | 1.24 | 26.8% | 15.6% | 7.5% | 0.22 | 0.10 | 82 |
| Cigarette | 22.28 | 1.44 | 0.06 | 22.22 | 9.82 | 0.52 | 0.0% | 0.8% | 2.2% | 0.20 | 0.03 | 202 |
| Coffee | 5.19 | 0.67 | 0.12 | 14.50 | 10.19 | 0.68 | 5.2% | 2.9% | 5.0% | 0.15 | 0.04 | 86 |
| Coldcer | 3.45 | 0.61 | 0.18 | 70.70 | 127.58 | 1.68 | 4.0% | 18.1% | 14.1% | 0.24 | -0.36 | 125 |
| Deod | 2.66 | 0.19 | 0.07 | 6.94 | 4.42 | 0.65 | 4.1% | 5.2% | 4.4% | 0.09 | -0.07 | 126 |
| Factiss | 2.12 | 0.14 | 0.07 | 75.82 | 43.36 | 0.48 | 0.3% | 11.7% | 3.7% | 0.31 | 0.04 | 6 |
| Fzdinen | 2.04 | 0.31 | 0.15 | 43.79 | 58.50 | 1.31 | 5.3% | 23.7% | 14.9% | 0.16 | -0.10 | 87 |
| Frozen pizza | 3.44 | 0.31 | 0.09 | 31.17 | 28.92 | 0.94 | 8.9% | 9.1% | 8.9% | 0.13 | 0.08 | 147 |
| Household Cleaner | 2.48 | 0.16 | 0.06 | 29.92 | 10.63 | 0.39 | 0.3% | 3.6% | 3.0% | 0.15 | 0.02 | 18 |
| Hotdog | 3.99 | 0.67 | 0.19 | 68.63 | 110.50 | 1.41 | 13.2% | 15.6% | 12.3% | 0.16 | 0.02 | 35 |
| Laundry Detergent | 8.78 | 0.85 | 0.11 | 28.94 | 52.35 | 1.34 | 2.3% | 8.8% | 10.3% | 0.16 | -0.11 | 57 |
| Margarine/Butter | 1.95 | 0.21 | 0.12 | 71.36 | 57.56 | 0.66 | 0.1% | 6.3% | 8.0% | 0.18 | -0.03 | 36 |
| Mayonnaise | 2.97 | 0.21 | 0.07 | 79.74 | 29.69 | 0.41 | 3.0% | 0.4% | 2.1% | 0.39 | -0.20 | 22 |
| Milk | 2.45 | 0.16 | 0.07 | 222.26 | 49.37 | 0.38 | 2.1% | 1.8% | 2.7% | 0.35 | 0.01 | 30 |
| Mustard & Ketchup | 2.06 | 0.23 | 0.12 | 64.51 | 57.32 | 0.61 | 5.3% | 0.9% | 3.2% | 0.33 | -0.03 | 22 |
| Paptowl | 3.66 | 0.46 | 0.09 | 68.07 | 211.54 | 2.81 | 4.0% | 3.6% | 8.3% | 0.37 | -0.35 | 3 |
| Peanut butter | 3.67 | 0.32 | 0.10 | 34.23 | 19.01 | 0.42 | 3.2% | 0.6% | 2.2% | 0.27 | -0.21 | 15 |
| Photo | 7.18 | 0.97 | 0.12 | 9.19 | 6.85 | 0.71 | 4.6% | 5.1% | 4.3% | 0.27 | -0.19 | 13 |
| Razors | 5.60 | 0.33 | 0.06 | 7.99 | 6.07 | 0.66 | 22.6% | 2.1% | 3.6% | 0.32 | -0.24 | 4 |
| Salty snacks | 2.28 | 0.28 | 0.13 | 50.89 | 63.88 | 1.05 | 6.7% | 5.0% | 6.8% | 0.21 | -0.18 | 100 |
| Shamp | 3.51 | 0.32 | 0.11 | 9.89 | 8.00 | 0.74 | 12.8% | 7.1% | 6.1% | 0.11 | -0.14 | 70 |
| Soup | 1.54 | 0.18 | 0.12 | 61.59 | 71.38 | 1.01 | 1.2% | 9.7% | 9.0% | 0.20 | -0.09 | 139 |
| Spagsau | 2.43 | 0.15 | 0.07 | 39.14 | 36.22 | 0.80 | 1.6% | 6.5% | 5.6% | 0.24 | 0.06 | 51 |
| Sugar substitutes | 2.76 | 0.22 | 0.08 | 14.49 | 7.44 | 0.53 | 0.1% | 1.4% | 4.1% | 0.15 | -0.03 | 20 |
| Toilet Tissue | 5.42 | 0.65 | 0.12 | 89.13 | 164.24 | 1.57 | 4.3% | 8.3% | 9.6% | 0.30 | -0.25 | 20 |
| Toothbrush | 2.56 | 0.28 | 0.11 | 8.69 | 5.84 | 0.71 | 3.1% | 6.3% | 5.0% | 0.12 | -0.14 | 27 |
| Toothpaste | 2.77 | 0.26 | 0.10 | 35.49 | 65.65 | 1.42 | 11.0% | 12.5% | 12.4% | 0.10 | -0.03 | 25 |
| Yogurt | 1.13 | 0.07 | 0.07 | 115.07 | 48.15 | 0.43 | 0.7% | 6.3% | 3.6% | 0.25 | 0.21 | 75 |

Figure 7. Unit sales, price (in USD), and promotional events (feature and display) for an SKU in the Beer category.



1. **Models**

In this study, we include the base-lift method as a benchmark model. This method has been widely used by retailers to forecast product sales at the SKU level (Cooper et al., 1999; Huang et al., 2014). The method generates baseline forecasts using simple exponential smoothing method with data when there is no promotion for the focal product. It then makes adjustments for any incoming promotional event based on the lift effect by the most recent promotional event. This can be represented as follows:

where is the final forecast for week *t* by the base-lift method, is the baseline forecast for week , is the actual sales for the previous week when the focal product is not being promoted, is the parameter for the simple exponential smoothing model. The adjustment is calculated as the increased sales by the most recent promotional event for the focal product.

We also consider two autoregressive distributed lag (ADL) models which were proposed by Huang et al. (2014). The ADL model captures the dynamic effects of price reductions and promotional events with parsimonious specifications. The first ADL model is the ADL model initially constructed with the dynamic terms of the price and promotional information of the focal product (we refer this model as the ADL-own model thereafter). The model form can be presented as follows:

where:

is the log sales of the focal product at week

is the week number which captures the time trend

is the log price of the focal product at week

is the promotional index of the focal product at week

is the four-week-dummy variable  
 is the dummy variable for the calendar event at week . The dummy variable represents the week of the calendar event when , , and the week before the event if . takes the values from 1 to 9 representing all the calendar events *[[9]](#footnote-9)*

are the parameters  
 is the error term and we assume

is the order of the lags and is set to as 2.

We then reduce the model with the Least Absolute Shrinkage and Selection Operator (LASSO) algorithm following Ma et al. (2016). The LASSO algorithm is a regularization algorithm which put a constraint to the sum of the absolute values of all the parameter coefficients of the initial ADL model. That is,

where

is the vector of observations on the dependent variable  
 represents explanatory variables included in the initial ADL model

*u* is the identically distributed random error

is the vector of the parameter coefficients  
*N* is the number of parameters  
 is the shrinkage factor which equals to the sum of all the parameter coefficients.

The initial model will be reduced when some of the parameter coefficients are pushed to zeros by the constraint. The model reduction process is controlled by a shrinkage factor which is determined by using 10-fold cross-validation following Ma et al. (2016). The modelling procedure is referred as the ADL-own model which can be illustrated in Figure 8a.

The other ADL model is the ADL model initially specified with the price and promotional information of the focal products and all the other competitive products within the same product category. The model form can be presented as follows:

where

is the log price of competitive product at week .

is the promotional index of competitive product at week .

is the number of competitive price variables in the product category.

is the number of competitive promotional variables in the product category.

We reduce the model with the LASSO procedure and then combine the retained explanatory variables with the variables retained in the ADL-own model. We refer this model as the ADL-intra model thereafter and the modelling procedure can be illustrated in Figure 8b.

Figure 8a. An illustration for the ADL-own models

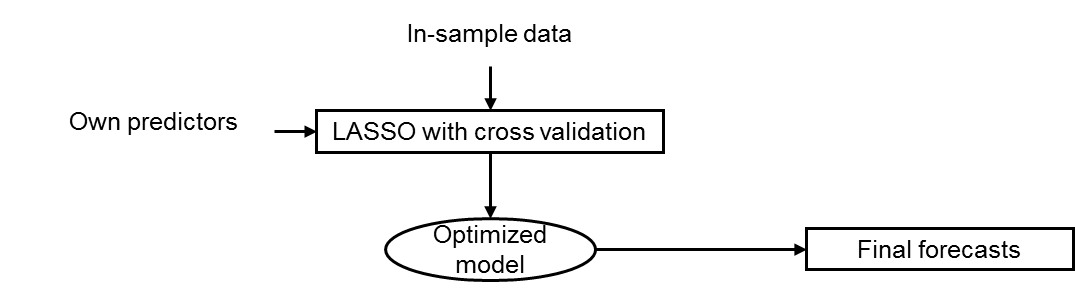


Figure 8b. An illustration for the ADL-intra models

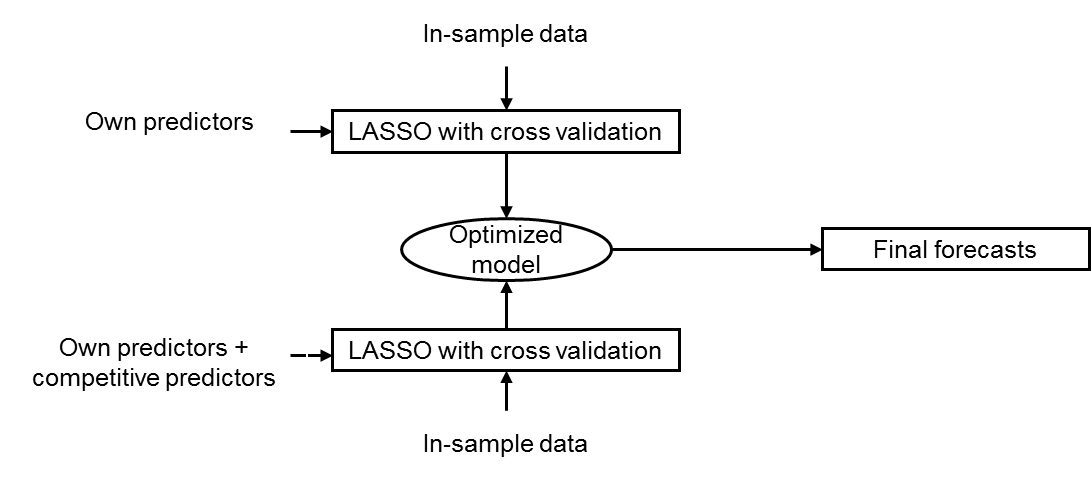


Figure 8c. An illustration for the ADL-intra-IC models

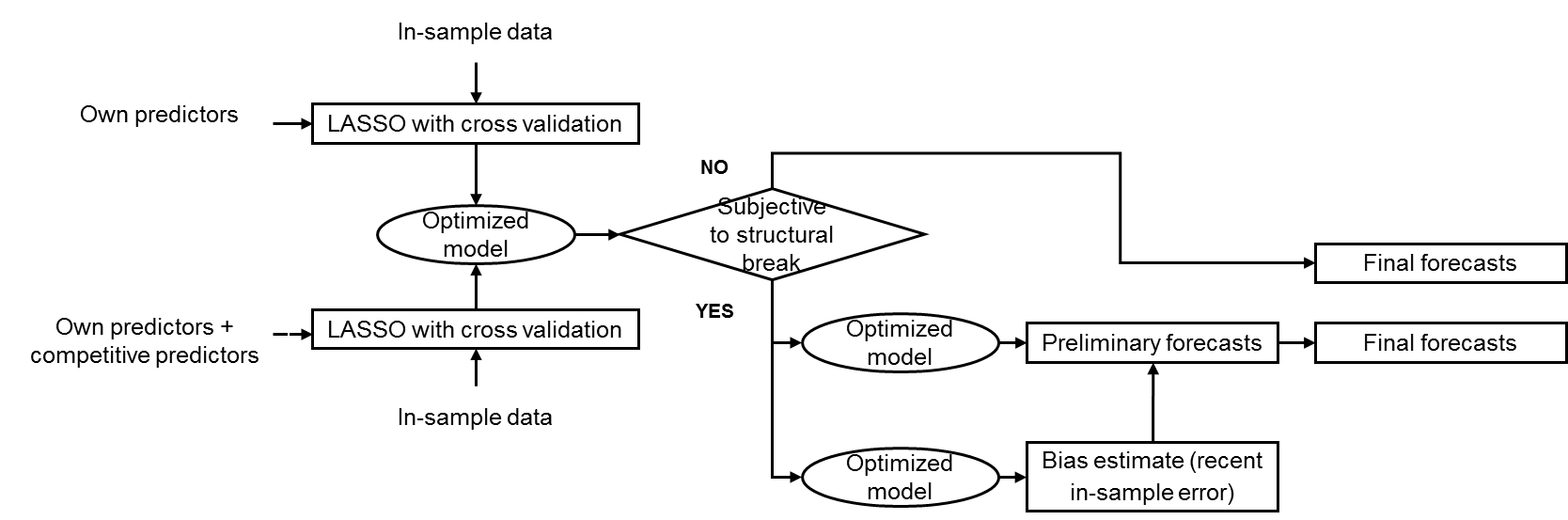
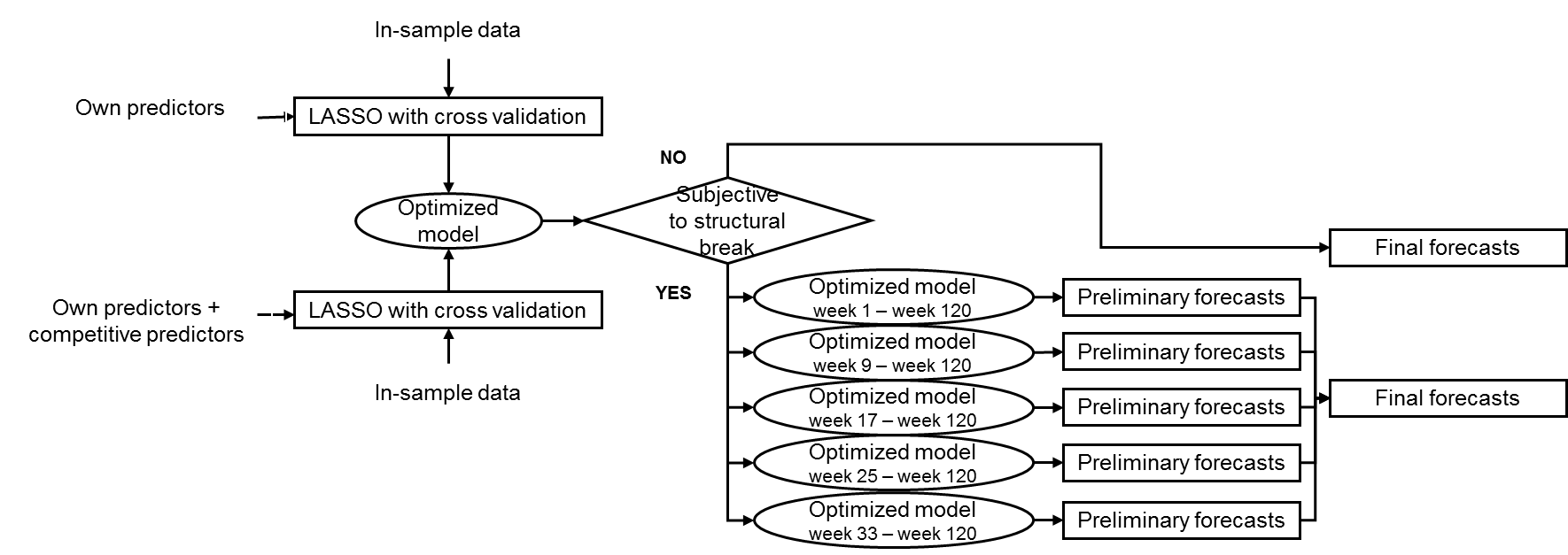


Figure 8d. An illustration for the ADL-intra-EWC models



The ADL-own model and the ADL-intra model implicitly assume that the effect of the price and promotional events of the focal product to be constant over time and may be potentially subject to structural break. In this study, we implement the intercept correction method and the estimation window combining method respectively on the two ADL models. The modelling procedure for the ADL-intra-EWC and the ADL-intra-IC model are illustrated in Figure 8c and 8d respectively. The benchmark and the candidate models are described introduced in Table 1.

|  |  |
| --- | --- |
| Model | Description |
| Base-lift | A two-stage method widely used in the industry |
| ADL-own | The ADL model with the promotional variables of the focal product only and then simplified by the LASSO algorithm. |
| ADL-intra | The ADL model with the promotional variables of the focal product only and all the competitive products within the same product category. The model is then simplified by the LASSO algorithm. and then the model will include the variables retained by the ADL-own model. |
| ADL-own-EWC | ADL-own model implemented with Estimation Window Combining |
| ADL-own-IC | ADL-own model implemented with Intercept Correction |
| ADL-intra-IC | ADL model implemented with Intercept Correction |
| ADL-intra-EWC | ADL model implemented with Estimation Window Combining |

In this study, we implement the intercept correction method and the estimation window combining method discriminately based on the results of a sequential Chow test for the identification of structural break[[10]](#footnote-10), as described in section 3. For the intercept correction method, we estimate the forecast bias based on an equally weighted average of four predictive errors before the forecast origin. Also, there are different correction schemes for the forecast bias when the model contains lagged product sales as explanatory variables. Different correction schemes lead to slightly different characteristics regarding bias reduction and forecast error variance inflation ([Clements and Hendry 1999](#_ENREF_15)). For the estimation window combining method, we adopt a combining scheme of equal weights as equal weighting scheme has been proved to be effective and easy to implement (Pesaran and Timmerman, 2005).

1. **The experimental design**

In this study, we evaluate the forecasting performance of the models with five rolling origins ([Tashman 2000](#_ENREF_65)). In Huang et al. (2016), the model estimation was updated in a rolling manner but the model specification was done with all the available data with a limitation of presuming prior knowledge of the data ([Fildes, Wei et al. 2011](#_ENREF_27)). In this study, we follow Ma and Fildes (2016) and re-specify the model for each rolling event. For example, we initially specify the models with the data from week 1 to week 120. we then re-specify the model respectively with updated data from week 9 to week 130, from week 17 to week 138, from week 25 to week 145, and from week 33 to week 153. For each rolling event, we generate the forecasts of one to weeks ahead, where is 1, 4, and 12. Therefore, our evaluation truly represents the situation retailers face in practice. We use the actual values of the exogenous variables (e.g., price, promotion, or calendar events etc.) and the forecasts of the lagged dependent variables when the forecast horizon is beyond one week.

We evaluate the models’ forecasting performance using four error measures: the Mean Absolute Percentage Error (MAPE), the symmetric Mean Absolute Percentage Error (sMAPE), the Mean Absolute Scaled Error (MASE) ([Hyndman and Koehler 2006](#_ENREF_35)), and the Relative Average Mean Absolute Error (RelAvgMAE) ([Davydenko and Fildes 2013](#_ENREF_20)). These error measures approximate the loss function of the retailer from different aspects. The error measures for SKUs and rolling events based on forecast horizon of 1 to (i.e. , , and =1, 4 and 12) are as follows:

where is the actual value in the forecast period for data series based on the rolling event. is the forecast value for data series based on the rolling event[[11]](#footnote-11). is the total number of observations in the full estimation window.

1. **Results and discussion**

8.1 results for all the forecast period across categories

Table 4a shows the forecasting performance of the candidate models for all the forecast period. The Base-lift model generate the least accurate forecasts for almost all the scenarios. The ADL-own model also gets outperformed by the ADL-intra model for all the scenarios, which highlights the value of competitive promotional information as suggested by Huang et al. (2014). In practice, competitive promotional information may not available for some manufacturers, and we may implement the EWC method and the IC method based on the ADL-own model. The ADL-own-EWC model outperforms the ADL-own model for all the scenarios. The ADL-own-IC model has mixed forecasting performance compared to the ADL-own model: it has superior forecasting performance for short forecast horizons (e.g., when *h*=1 and *h*=4) but was losing advantages when forecast horizons get longer (e.g., when *h*=12). When competitive promotional information become available, we implement the EWC method and the IC method based on the ADL-intra model. The ADL-intra-EWC model outperforms the ADL-intra model for all the scenarios. The ADL-intra-IC model has mixed forecasting performance compared the ADL-intra model. The ADL-intra-IC model has superior forecasting performance for 1-week-ahead and 4-week-ahead forecast horizons but gets outperformed for 12-week-ahead forecast horizon. The forecasting performance are consistent across different error measures. Overall, the ADL-intra-EWC model and the ADL-intra-IC model are the most accurate models. The ADL-intra-EWC model has the best forecasting performance for 12-week-ahead forecast horizon, while the ADL-intra-IC model has the best forecasting performance for 1-week-ahead forecast horizon. They have comparable forecasting performance for 4-week-ahead forecast horizon.

Table 3a. The forecasting performance of candidate models for all forecast period for different forecast horizons

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| All forecast period, Forecast horizon= 12 | | | | | | | | |
| Model/measure | MAPE | Rank | SMAPE | Rank | MASE | Rank | AvgRelMAE | Rank |
| Base-lift | 71.17% | 7 | 47.55% | 7 | 0.783 | 7 | 1.1458 | 7 |
| ADL-own | 67.99% | 5 | 41.16% | 5 | 0.700 | 5 | 1.0000 | 5 |
| ADL-intra | 67.00% | 2 | 40.84% | 2 | 0.696 | 3 | 0.9925 | 2 |
| ADL-own-EWC | 67.86% | 4 | 40.95% | 3 | 0.696 | 2 | 0.9932 | 3 |
| ADL-intra-EWC | 66.87% | 1 | 40.64% | 1 | 0.691 | 1 | 0.9856 | 1 |
| ADL-own-IC | 68.34% | 6 | 41.39% | 6 | 0.704 | 6 | 1.0031 | 6 |
| ADL-intra-IC | 67.27% | 3 | 41.05% | 4 | 0.700 | 4 | 0.9958 | 4 |
| All forecast period, Forecast horizon= 4 | | | | | | | | |
| Model/measure | MAPE | Rank | SMAPE | Rank | MASE | Rank | AvgRelMAE | Rank |
| Base-lift | 66.41% | 7 | 45.94% | 7 | 0.751 | 7 | 1.0944 | 7 |
| ADL-own | 65.73% | 6 | 40.53% | 6 | 0.688 | 6 | 1.0000 | 6 |
| ADL-intra | 64.88% | 3 | 40.18% | 3 | 0.680 | 3 | 0.9907 | 3 |
| ADL-own-EWC | 65.54% | 5 | 40.31% | 4 | 0.683 | 5 | 0.9925 | 4 |
| ADL-intra-EWC | 64.70% | 2 | 39.96% | 1 | 0.676 | 1 | 0.9834 | 1 |
| ADL-own-IC | 65.15% | 4 | 40.47% | 5 | 0.682 | 4 | 0.9955 | 5 |
| ADL-intra-IC | 64.46% | 1 | 40.12% | 2 | 0.677 | 2 | 0.9866 | 2 |
| All forecast period, Forecast horizon= 1 | | | | | | | | |
| Model/measure | MAPE | Rank | SMAPE | Rank | MASE | Rank | AvgRelMAE | Rank |
| Base-lift | 61.82% | 6 | 44.06% | 7 | 0.726 | 7 | 1.0062 | 7 |
| ADL-own | 62.04% | 7 | 39.53% | 6 | 0.666 | 6 | 1.0000 | 6 |
| ADL-intra | 61.44% | 4 | 39.22% | 4 | 0.657 | 4 | 0.9974 | 5 |
| ADL-own-EWC | 61.71% | 5 | 39.33% | 5 | 0.663 | 5 | 0.9954 | 4 |
| ADL-intra-EWC | 60.93% | 3 | 39.02% | 2 | 0.654 | 2 | 0.9836 | 3 |
| ADL-own-IC | 60.46% | 2 | 39.09% | 3 | 0.656 | 3 | 0.9707 | 2 |
| ADL-intra-IC | 60.15% | 1 | 38.82% | 1 | 0.650 | 1 | 0.9677 | 1 |

8.2 results for promoted forecast period and non-promoted forecast period across categories

Table 4b and Table 4c respectively shows the forecasting performance of the candidate models for the forecast period when the focal product is being promoted with either feature or display and when the focal product is not being promoted[[12]](#footnote-12). The results are overall in consistent with the results for all the forecast period described in section 8.1. For the promoted period, the Base-lift model has the least accurate forecasts but surprisingly have good performance for MAPE for 4-week-ahead and 12-week-ahead forecast horizons. The ADL-own model again gets outperformed by the ADL-intra model for all the scenarios. The ADL-own-EWC model outperforms the ADL-own model for most of the scenarios. The ADL-own-IC model has mixed forecasting performance compared to the ADL-own model: it outperforms the ADL-own model for short forecast horizons (e.g., when *h*=1). The ADL-intra-EWC model outperforms the ADL-intra model for most of the scenarios. The ADL-intra-IC model has superior forecasting performance for 1-week-ahead forecast horizon and mixed forecasting performance compared to the ADL-intra model, and gets outperformed by the ADL-intra model for the 12-week-ahead forecast horizon.

For the non-promoted period, as shown in Table 4c, the Base-lift model has the least accurate forecasts for most error measures except for the MASE and the AvgRelMAE for 1-week-ahead forecast horizon. The ADL-own model is outperformed by the ADL-intra model for all the scenarios. The ADL-own-EWC model outperforms the ADL-own model for all the scenarios. The ADL-own-IC model outperforms the ADL-own model for short and middle forecast horizons (e.g., when *h*=1 or 4) and has mixed forecasting performance with the ADL-own model for 12-week-ahead forecast horizon. The ADL-intra-EWC model outperforms the ADL-intra model for all the scenarios. The ADL-intra-IC model has superior forecasting performance for 1-week-ahead and 4-week-ahead forecast horizons and mixed forecasting performance compared to the ADL-intra model for 12-week-ahead forecast horizon.

Table 3b. The forecasting performance of candidate models for the promoted forecast period for different forecast horizons

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| promoted period, Forecast horizon= 12 | | | | | | | | |
| Model/measure | MAPE | Rank | SMAPE | Rank | MASE | Rank | AvgRelMAE | Rank |
| Base-lift | 66.36% | 1 | 81.78% | 7 | 2.162 | 7 | 1.4518 | 7 |
| ADL-own | 74.93% | 4 | 49.89% | 5 | 1.656 | 4 | 1.0000 | 5 |
| ADL-intra | 73.89% | 3 | 48.82% | 2 | 1.633 | 2 | 0.9769 | 2 |
| ADL-own-EWC | 74.97% | 5 | 49.56% | 4 | 1.645 | 3 | 0.9906 | 3 |
| ADL-intra-EWC | 73.58% | 2 | 48.42% | 1 | 1.616 | 1 | 0.9659 | 1 |
| ADL-own-IC | 76.98% | 7 | 50.56% | 6 | 1.696 | 6 | 1.0265 | 6 |
| ADL-intra-IC | 75.35% | 6 | 49.28% | 3 | 1.668 | 5 | 0.9962 | 4 |
| promoted period, Forecast horizon= 4 | | | | | | | | |
| Model/measure | MAPE | Rank | SMAPE | Rank | MASE | Rank | AvgRelMAE | Rank |
| Base-lift | 64.18% | 1 | 81.35% | 7 | 2.107 | 7 | 1.5123 | 7 |
| ADL-own | 73.50% | 6 | 50.05% | 5 | 1.617 | 6 | 1.0000 | 5 |
| ADL-intra | 72.02% | 3 | 48.46% | 2 | 1.560 | 2 | 0.9676 | 2 |
| ADL-own-EWC | 74.84% | 7 | 49.82% | 4 | 1.605 | 4 | 0.9897 | 4 |
| ADL-intra-EWC | 73.26% | 5 | 48.30% | 1 | 1.548 | 1 | 0.9593 | 1 |
| ADL-own-IC | 73.18% | 4 | 50.41% | 6 | 1.612 | 5 | 1.0131 | 6 |
| ADL-intra-IC | 71.92% | 2 | 48.65% | 3 | 1.563 | 3 | 0.9753 | 3 |
| promoted period, Forecast horizon= 1 | | | | | | | | |
| Model/measure | MAPE | Rank | SMAPE | Rank | MASE | Rank | AvgRelMAE | Rank |
| Base-lift | 66.42% | 4 | 87.12% | 7 | 2.220 | 7 | 1.4461 | 7 |
| ADL-own | 67.11% | 7 | 50.14% | 5 | 1.603 | 5 | 1.0000 | 5 |
| ADL-intra | 66.63% | 5 | 48.56% | 3 | 1.510 | 1 | 0.9792 | 3 |
| ADL-own-EWC | 66.79% | 6 | 50.00% | 4 | 1.605 | 6 | 1.0044 | 6 |
| ADL-intra-EWC | 65.85% | 2 | 48.47% | 2 | 1.516 | 3 | 0.9715 | 2 |
| ADL-own-IC | 66.39% | 3 | 50.20% | 6 | 1.587 | 4 | 0.9902 | 4 |
| ADL-intra-IC | 65.71% | 1 | 48.30% | 1 | 1.511 | 2 | 0.9523 | 1 |

Table 3c. The forecasting performance of candidate models for the non-promoted forecast period for different forecast horizons

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| non-promoted period, Forecast horizon= 12 | | | | | | | | |
| Model/measure | MAPE | Rank | SMAPE | Rank | MASE | Rank | AvgRelMAE | Rank |
| Base-lift | 72.83% | 7 | 42.44% | 7 | 0.600 | 7 | 1.0194 | 7 |
| ADL-own | 68.68% | 5 | 40.64% | 5 | 0.586 | 6 | 1.0000 | 5 |
| ADL-intra | 67.59% | 2 | 40.38% | 2 | 0.584 | 4 | 0.9940 | 3 |
| ADL-own-EWC | 68.47% | 4 | 40.44% | 3 | 0.581 | 2 | 0.9934 | 2 |
| ADL-intra-EWC | 67.37% | 1 | 40.20% | 1 | 0.579 | 1 | 0.9893 | 1 |
| ADL-own-IC | 68.82% | 6 | 40.83% | 6 | 0.585 | 5 | 1.0013 | 6 |
| ADL-intra-IC | 67.73% | 3 | 40.58% | 4 | 0.583 | 3 | 0.9978 | 4 |
| non-promoted period, Forecast horizon= 4 | | | | | | | | |
| Model/measure | MAPE | Rank | SMAPE | Rank | MASE | Rank | AvgRelMAE | Rank |
| Base-lift | 67.42% | 7 | 40.73% | 7 | 0.571 | 6 | 0.9894 | 1 |
| ADL-own | 66.19% | 6 | 39.88% | 6 | 0.572 | 7 | 1.0000 | 7 |
| ADL-intra | 65.23% | 3 | 39.63% | 3 | 0.570 | 5 | 0.9962 | 6 |
| ADL-own-EWC | 65.85% | 5 | 39.67% | 4 | 0.567 | 4 | 0.9925 | 4 |
| ADL-intra-EWC | 64.93% | 2 | 39.43% | 1 | 0.566 | 2 | 0.9895 | 2 |
| ADL-own-IC | 65.55% | 4 | 39.73% | 5 | 0.566 | 3 | 0.9940 | 5 |
| ADL-intra-IC | 64.77% | 1 | 39.52% | 2 | 0.566 | 1 | 0.9905 | 3 |
| non-promoted period, Forecast horizon= 1 | | | | | | | | |
| Model/measure | MAPE | Rank | SMAPE | Rank | MASE | Rank | AvgRelMAE | Rank |
| Base-lift | 62.10% | 7 | 38.69% | 7 | 0.543 | 1 | 0.9614 | 1 |
| ADL-own | 61.36% | 6 | 38.59% | 6 | 0.552 | 7 | 1.0000 | 7 |
| ADL-intra | 60.52% | 4 | 38.39% | 5 | 0.550 | 6 | 0.9993 | 6 |
| ADL-own-EWC | 61.01% | 5 | 38.34% | 4 | 0.547 | 5 | 0.9945 | 5 |
| ADL-intra-EWC | 60.05% | 3 | 38.14% | 3 | 0.545 | 4 | 0.9848 | 4 |
| ADL-own-IC | 59.73% | 2 | 38.13% | 2 | 0.544 | 2 | 0.9694 | 2 |
| ADL-intra-IC | 59.19% | 1 | 38.00% | 1 | 0.544 | 3 | 0.9706 | 3 |

8.3 comparable results for all forecast period for each product category

Results shown in section 8.2 indicate that the EWC method and the IC method generate more accurate forecasts across the 30 product categories. We further explore their forecasting performance for each individual product category. Table 5a shows the improvement in term of the MAPE based on different forecast horizons for the following models over their counterparts: 1) the ADL-intra model versus the ADL-own model; 2) the ADL-own-EWC model versus the ADL-own model; 3) the ADL-own-IC model versus the ADL-own model; 4) the ADL-intra-EWC model versus the ADL- intra model; 5) and the ADL- intra -IC model versus the ADL-intra model. In the table, the values represent the percentage reduction (highlighted in green) or increase of the MAPE by one model over the other. For example, compared to the ADL-own model, we could achieve a MAPE value which is 1.40% lower by using the ADL-intra model. The value of 1.40% for “ADL-own - ADL-intra” in Table 5a is calculated as

Table 5a explores the results comparison between each pair of models for the MAPE. The positive values are highlighted in green which may indicate that the ADL-intra model outperforms the ADL-own model in the “ADL-own - ADL-intra” column or the ADL-intra-EWC model outperforms the ADL-intra model in the “ADL-intra - ADL-intra-EWC” column, and so forth. Table 5a suggests that for most of the product categories, the comparable forecasting performance of the various candidate models are in consistent with the results found in section 8.1 and 8.2. For example, 1) the ADL-intra model outperforms the ADL-own model for most of the categories; 2) the ADL-own-EWC model and the ADL-own-IC model generally outperform the ADL-own model, and the ADL-intra-EWC model and the ADL- intra -IC model generally outperform the ADL- intra model; 3) the advantage of the ADL-own-IC model and the ADL-intra-IC model over their counterparts (e.g., the ADL-own model and the ADL-intra model) are getting marginal as the forecast horizon increases. There are also some exceptional product categories including Carbonated Beverages, Frozen pizza, Hotdog, Razors, Margarine/Butter, Toothbrush, and Salty snacks where the ADL models combined with the EWC method and the IC method get outperformed by the ADL-own model or the ADL-intra model for most of the scenarios (e.g., error measures and forecast horizons). Table 5b and Table 5c have same indications for the MASE and the SMAPE. Table 5d shows the values of AvgRelMAE for the various pairs of models. The table either shows some of the original values of the AvgRelMAE (e.g., for the columns of “ADL-own - ADL-intra”, “ADL-own - ADL-own-EWC”, “ADL-own - ADL-own-IC”) which calculated based on the ADL-own model. The values below 1 are highlighted in green and indicate that the candidate models outperforms the ADL-own model. The table also shows the difference of the AvgRelMAE value for some other models. For example, the values in the column “ADL-intra versus ADL-intra-EWC” are calculated as , and the values in the column “ADL-intra versus ADL-intra-IC” are calculated as . The positive values are highlighted in green.

Table 4a Comparing forecasting performance for each product category for the MAPE.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Category/ MAPE | Horizon=1 | | | | | Horizon=4 | | | | | Horizon=12 | | | | |
| ADL-own - ADL-intra | ADL-own - ADL-own-EWC | ADL-own - ADL-own-IC | ADL-intra - ADL-intra-EWC | ADL-intra - ADL-intra-IC | ADL-own - ADL-intra | ADL-own - ADL-own-EWC | ADL-own - ADL-own-IC | ADL-intra - ADL-intra-EWC | ADL-intra - ADL-intra-IC | ADL-own - ADL-intra | ADL-own - ADL-own-EWC | ADL-own - ADL-own-IC | ADL-intra - ADL-intra-EWC | ADL-intra - ADL-intra-IC |
| Paptowl | 1.40% | -6.26% | 18.01% | -5.61% | 18.51% | 1.40% | -6.26% | 18.01% | -5.61% | 18.51% | 0.80% | -7.03% | 5.66% | -7.55% | 5.64% |
| Beer | 0.76% | 0.57% | 0.54% | 0.59% | 0.65% | 0.76% | 0.57% | 0.54% | 0.59% | 0.65% | 0.81% | 0.89% | -0.68% | 0.93% | -0.27% |
| Blades | -0.20% | 1.15% | -3.96% | 0.26% | -4.39% | -0.20% | 1.15% | -3.96% | 0.26% | -4.39% | 0.11% | 2.69% | 3.12% | 2.39% | 2.28% |
| Carbonated Beverages | 4.28% | -3.24% | -7.76% | -2.77% | -8.64% | 4.28% | -3.24% | -7.76% | -2.77% | -8.64% | 3.13% | -3.02% | -9.64% | -2.40% | -9.44% |
| Cigarette | -0.57% | 0.36% | -1.70% | 0.48% | -1.43% | -0.57% | 0.36% | -1.70% | 0.48% | -1.43% | -0.09% | 0.01% | -1.84% | -0.20% | -1.79% |
| Coffee | 0.80% | -0.39% | 7.74% | -0.44% | 6.52% | 0.80% | -0.39% | 7.74% | -0.44% | 6.52% | 0.87% | -0.91% | 3.14% | -0.79% | 3.16% |
| Coldcer | -2.28% | -0.54% | -2.90% | 1.05% | -2.92% | -2.28% | -0.54% | -2.90% | 1.05% | -2.92% | 0.28% | -2.39% | -6.96% | -1.88% | -6.85% |
| Deod | 0.74% | 0.87% | 3.34% | 1.04% | 3.50% | 0.74% | 0.87% | 3.34% | 1.04% | 3.50% | 0.85% | 0.75% | 3.67% | 0.74% | 3.65% |
| Factiss | -1.87% | 8.10% | 0.39% | 6.80% | 5.44% | -1.87% | 8.10% | 0.39% | 6.80% | 5.44% | 0.42% | 8.28% | -7.03% | 7.17% | -8.92% |
| Fzdinen | 2.60% | 2.75% | 15.27% | 3.19% | 12.72% | 2.60% | 2.75% | 15.27% | 3.19% | 12.72% | 3.17% | 2.68% | 7.32% | 2.22% | 6.54% |
| Frozen pizza | -0.62% | -2.58% | -6.15% | -1.69% | -5.06% | -0.62% | -2.58% | -6.15% | -1.69% | -5.06% | 0.94% | -2.17% | -6.90% | -2.27% | -6.19% |
| Household Cleaner | 2.89% | 2.26% | 7.35% | 1.70% | 4.87% | 2.89% | 2.26% | 7.35% | 1.70% | 4.87% | -0.48% | 3.45% | -0.24% | 3.44% | -0.77% |
| Hotdog | -2.24% | 1.35% | -6.24% | 2.17% | -4.49% | -2.24% | 1.35% | -6.24% | 2.17% | -4.49% | 1.58% | -3.50% | -13.66% | -3.16% | -11.69% |
| Laundry Detergent | 0.28% | 1.80% | 2.47% | 1.80% | 3.43% | 0.28% | 1.80% | 2.47% | 1.80% | 3.43% | 0.86% | 1.73% | -4.22% | 1.88% | -1.98% |
| Margarine/Butter | -1.39% | -0.64% | 3.19% | -0.94% | 3.43% | -1.39% | -0.64% | 3.19% | -0.94% | 3.43% | -0.11% | -0.46% | -1.28% | -0.72% | -1.68% |
| Mayonnaise | 1.50% | 1.71% | 5.79% | 1.19% | 5.53% | 1.50% | 1.71% | 5.79% | 1.19% | 5.53% | 0.93% | 0.94% | -6.24% | 0.62% | -6.16% |
| Milk | 0.64% | 3.43% | 6.14% | 3.01% | 5.89% | 0.64% | 3.43% | 6.14% | 3.01% | 5.89% | -1.17% | 4.16% | 7.82% | 3.90% | 7.62% |
| Mustard & Ketchup | -1.78% | 3.31% | 3.04% | 3.36% | 4.65% | -1.78% | 3.31% | 3.04% | 3.36% | 4.65% | 0.42% | 2.61% | 1.36% | 2.51% | 1.00% |
| Peanut butter | 0.14% | -0.83% | -2.52% | -1.60% | -2.49% | 0.14% | -0.83% | -2.52% | -1.60% | -2.49% | 0.30% | 3.22% | -2.14% | 3.05% | -2.48% |
| Photo | -0.16% | 2.21% | 8.54% | 1.86% | 7.79% | -0.16% | 2.21% | 8.54% | 1.86% | 7.79% | 0.10% | 2.12% | 4.12% | 1.84% | 4.62% |
| Razors | 0.00% | -1.74% | 3.38% | -1.74% | 3.38% | 0.00% | -1.74% | 3.38% | -1.74% | 3.38% | 0.00% | -3.85% | -1.98% | -3.85% | -1.98% |
| Salty snacks | -0.55% | -0.29% | 3.26% | -0.25% | 4.24% | -0.55% | -0.29% | 3.26% | -0.25% | 4.24% | -0.16% | -1.40% | -0.82% | -1.51% | -0.22% |
| Shamp | 1.21% | 1.97% | 4.91% | 1.47% | 3.78% | 1.21% | 1.97% | 4.91% | 1.47% | 3.78% | 1.61% | 2.86% | 5.34% | 2.61% | 4.66% |
| Soup | 7.06% | 1.70% | 8.78% | 1.19% | 5.52% | 7.06% | 1.70% | 8.78% | 1.19% | 5.52% | 4.59% | 1.23% | 0.86% | 1.09% | -0.19% |
| Spagsau | 2.76% | 8.66% | 8.37% | 6.81% | 6.79% | 2.76% | 8.66% | 8.37% | 6.81% | 6.79% | 1.88% | 8.66% | 4.36% | 7.87% | 3.92% |
| Sugar substitutes | 0.84% | 0.58% | 4.89% | 0.81% | 4.19% | 0.84% | 0.58% | 4.89% | 0.81% | 4.19% | 1.22% | 0.74% | 4.35% | 0.63% | 2.58% |
| Toilet Tissue | -0.37% | -0.62% | 8.45% | 1.24% | 11.86% | -0.37% | -0.62% | 8.45% | 1.24% | 11.86% | -1.59% | -0.13% | 2.73% | 0.47% | 4.54% |
| Toothbrush | 2.03% | -1.97% | -1.24% | -1.80% | -1.66% | 2.03% | -1.97% | -1.24% | -1.80% | -1.66% | 0.33% | -2.29% | -3.85% | -1.68% | -2.83% |
| Toothpaste | 0.65% | -1.59% | 2.83% | 1.66% | 2.48% | 0.65% | -1.59% | 2.83% | 1.66% | 2.48% | 5.72% | -1.09% | 3.96% | -0.46% | 5.25% |
| Yogurt | 0.16% | 3.52% | 5.99% | 3.30% | 4.75% | 0.16% | 3.52% | 5.99% | 3.30% | 4.75% | 0.60% | 1.03% | 2.20% | 0.65% | 1.04% |

Table 4b Comparing forecasting performance for each product category for the MASE.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Category/MASE | Horizon=1 | | | | | | Horizon=4 | | | | | | Horizon=12 | | | | | |
| ADL-own - ADL-intra | ADL-own - ADL-own-EWC | ADL-own - ADL-own-IC | ADL-intra - ADL-intra-EWC | ADL-intra - ADL-intra-IC | ADL-own - ADL-intra | | ADL-own - ADL-own-EWC | ADL-own - ADL-own-IC | ADL-intra - ADL-intra-EWC | ADL-intra - ADL-intra-IC | ADL-own - ADL-intra | | ADL-own - ADL-own-EWC | ADL-own - ADL-own-IC | ADL-intra - ADL-intra-EWC | ADL-intra - ADL-intra-IC |
| Paptowl | 2.18% | 10.03% | 1.54% | 12.01% | 2.22% | 2.32% | | 4.72% | 5.27% | 5.24% | 4.69% | 0.93% | | -4.43% | 9.85% | -4.26% | 9.85% |
| Beer | 2.13% | 0.03% | 0.33% | -0.17% | -0.66% | 1.00% | | 0.65% | -0.20% | 0.63% | -0.38% | 0.86% | | 0.59% | -1.07% | 0.55% | -1.08% |
| Blades | 0.46% | 0.93% | -1.07% | 0.02% | -1.93% | 1.42% | | 1.58% | 2.69% | 1.06% | 1.14% | 0.89% | | 2.01% | 2.95% | 1.79% | 1.38% |
| Carbonated Beverages | 2.58% | 0.06% | 2.05% | 0.83% | 1.40% | 0.49% | | 0.16% | 0.28% | 0.15% | -0.22% | 0.33% | | -0.10% | -1.16% | -0.05% | -1.27% |
| Cigarette | -0.50% | 0.22% | 0.39% | 0.47% | 0.31% | -0.13% | | 0.93% | 1.00% | 1.10% | 0.77% | -0.27% | | 0.49% | 1.16% | 0.55% | 1.20% |
| Coffee | 0.33% | -0.18% | 3.60% | -0.35% | 2.38% | 0.33% | | 0.12% | -0.22% | 0.02% | -0.63% | 0.24% | | -0.14% | -0.39% | -0.05% | -0.05% |
| Coldcer | 2.18% | -1.43% | 1.52% | -0.25% | 1.28% | 0.76% | | -0.96% | -3.40% | -0.28% | -3.36% | 0.53% | | -0.92% | -2.75% | -0.49% | -2.05% |
| Deod | 0.03% | 0.86% | 0.87% | 1.02% | 1.25% | 1.19% | | 0.69% | 1.23% | 0.62% | 0.94% | 0.84% | | 0.47% | 1.10% | 0.48% | 1.05% |
| Factiss | -0.06% | 5.97% | -4.64% | 4.81% | -1.27% | 0.11% | | 3.73% | -3.95% | 2.06% | -3.74% | -0.05% | | 4.37% | -5.24% | 2.85% | -6.04% |
| Fzdinen | 13.90% | 1.53% | 4.30% | 1.62% | -2.44% | 11.82% | | 0.24% | 1.74% | 0.00% | -3.42% | 4.77% | | 0.28% | -3.60% | -0.29% | -4.58% |
| Frozen pizza | -0.42% | -0.56% | -2.31% | -0.27% | -0.98% | 0.16% | | -0.47% | -0.93% | -1.43% | -0.60% | 1.04% | | -0.49% | -4.02% | -0.71% | -3.66% |
| Household Cleaner | 2.33% | 1.09% | 6.72% | 0.60% | 4.49% | 0.30% | | 1.32% | 4.24% | 1.53% | 3.57% | -0.62% | | 2.18% | 0.98% | 2.23% | 0.83% |
| Hotdog | 0.17% | 1.21% | -5.30% | 1.52% | -2.67% | 0.08% | | -4.13% | -5.43% | -3.57% | -4.63% | 0.22% | | -3.91% | -8.80% | -4.15% | -8.35% |
| Laundry Detergent | 1.36% | 0.69% | 3.69% | 0.26% | 3.89% | 1.67% | | 1.23% | 1.36% | 1.50% | 2.20% | 0.85% | | 1.20% | -1.33% | 1.46% | 0.46% |
| Margarine/Butter | -3.20% | -1.23% | -3.45% | -1.74% | -3.81% | -1.37% | | -1.36% | -1.93% | -1.54% | -1.22% | -1.06% | | -1.52% | -4.77% | -1.41% | -4.93% |
| Mayonnaise | 1.86% | 1.51% | 8.94% | 0.61% | 8.34% | -0.34% | | 0.11% | 2.78% | -0.18% | 1.82% | 0.22% | | 0.47% | -2.57% | 0.18% | -3.02% |
| Milk | 2.65% | 1.92% | 6.21% | 1.68% | 4.55% | 0.25% | | 1.89% | 5.66% | 1.59% | 4.69% | 0.09% | | 2.28% | 5.88% | 2.20% | 4.88% |
| Mustard & Ketchup | -0.67% | 1.92% | 1.27% | 2.75% | 1.81% | 0.01% | | 0.85% | -0.88% | 0.73% | -0.83% | 0.05% | | 1.16% | -1.46% | 1.25% | -2.22% |
| Peanut butter | -0.49% | -3.21% | 5.57% | -3.69% | 5.28% | 0.17% | | 0.32% | 6.49% | 0.21% | 5.93% | 0.30% | | 0.18% | 4.32% | 0.12% | 3.70% |
| Photo | -0.67% | 2.61% | 6.59% | 2.43% | 5.74% | 0.34% | | 2.32% | 3.76% | 2.00% | 4.00% | 0.30% | | 1.81% | 0.47% | 1.66% | 0.59% |
| Razors | 0.00% | -2.68% | -0.36% | -2.68% | -0.36% | 0.00% | | -1.96% | 0.58% | -1.96% | 0.58% | 0.00% | | -1.45% | -3.56% | -1.45% | -3.56% |
| Salty snacks | 0.09% | 1.89% | 2.16% | 1.80% | 2.68% | 2.29% | | 0.62% | -0.58% | 0.43% | -0.97% | 1.19% | | 0.82% | -0.58% | 0.67% | -0.76% |
| Shamp | 1.65% | 0.87% | 2.80% | 0.60% | 2.48% | -0.26% | | 1.07% | 1.38% | 0.93% | 1.15% | 0.66% | | 1.12% | 0.81% | 0.97% | 0.19% |
| Soup | 4.09% | 1.63% | 1.29% | 1.09% | 0.28% | 3.86% | | 1.91% | 0.62% | 1.38% | 0.66% | 1.16% | | 1.30% | -4.04% | 1.81% | -2.89% |
| Spagsau | 4.36% | 2.82% | 3.72% | 0.13% | 2.93% | -1.03% | | 5.31% | 0.52% | 5.13% | 0.37% | -0.26% | | 5.41% | 0.81% | 6.14% | 0.77% |
| Sugar substitutes | 0.91% | 0.73% | 5.01% | 0.92% | 4.80% | 0.82% | | 0.60% | 7.04% | 1.03% | 6.73% | 2.01% | | 1.21% | 6.65% | 1.39% | 4.97% |
| Toilet Tissue | -0.36% | -6.24% | -3.86% | -8.36% | -3.31% | 0.45% | | -2.44% | 4.04% | -3.34% | 3.80% | -0.55% | | -1.99% | 1.11% | -1.88% | 1.19% |
| Toothbrush | 5.13% | -0.25% | -4.82% | -0.44% | -5.06% | 0.99% | | -1.34% | -4.26% | -1.10% | -4.38% | 0.78% | | -1.02% | -4.53% | -0.67% | -4.21% |
| Toothpaste | -5.22% | -1.06% | 1.11% | 0.64% | 0.50% | -0.54% | | -2.21% | -1.30% | -1.43% | -0.16% | -1.42% | | -1.21% | -4.73% | 0.25% | -1.81% |
| Yogurt | -0.69% | 3.36% | 6.45% | 3.02% | 4.66% | 0.49% | | 3.87% | 6.52% | 3.51% | 5.46% | 0.54% | | 4.00% | 5.63% | 3.69% | 4.08% |

Table 4c Comparing forecasting performance for each product category for the SMAPE.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Category/SMAPE | Horizon=1 | | | | | Horizon=4 | | | | | Horizon=12 | | | | |
| ADL-own - ADL-intra | ADL-own - ADL-own-EWC | ADL-own - ADL-own-IC | ADL-intra - ADL-intra-EWC | ADL-intra - ADL-intra-IC | ADL-own - ADL-intra | ADL-own - ADL-own-EWC | ADL-own - ADL-own-IC | ADL-intra - ADL-intra-EWC | ADL-intra - ADL-intra-IC | ADL-own - ADL-intra | ADL-own - ADL-own-EWC | ADL-own - ADL-own-IC | ADL-intra - ADL-intra-EWC | ADL-intra - ADL-intra-IC |
| Paptowl | 1.98% | -0.57% | 14.78% | 1.17% | 15.23% | 1.31% | -0.62% | 1.69% | -0.09% | 1.18% | 0.38% | -2.90% | 4.70% | -2.73% | 4.67% |
| Beer | 1.26% | 0.30% | 1.04% | 0.20% | 0.54% | 0.60% | 0.58% | 0.39% | 0.54% | 0.45% | 0.57% | 0.52% | -0.33% | 0.46% | -0.31% |
| Blades | -0.29% | -0.22% | -2.36% | -0.81% | -2.26% | 0.81% | 0.70% | 0.36% | 0.29% | -0.32% | 0.58% | 1.21% | 1.44% | 0.98% | 0.51% |
| Carbonated Beverages | 3.01% | -1.19% | -1.57% | -1.01% | -2.18% | 1.92% | -1.19% | -2.63% | -1.01% | -3.05% | 1.44% | -1.04% | -2.38% | -1.04% | -2.39% |
| Cigarette | -0.58% | 0.25% | 0.83% | 0.34% | 0.92% | -0.12% | 0.70% | 0.73% | 0.69% | 0.52% | -0.17% | 0.46% | 0.89% | 0.44% | 0.77% |
| Coffee | 0.16% | -0.26% | 3.65% | -0.29% | 2.64% | 0.59% | -0.22% | 0.78% | -0.15% | 0.30% | 0.47% | -0.37% | 0.25% | -0.27% | 0.02% |
| Coldcer | 1.43% | -1.05% | -1.36% | -0.83% | -1.80% | 0.97% | -0.86% | -2.84% | -0.61% | -2.34% | 0.60% | -0.68% | -3.06% | -0.35% | -2.37% |
| Deod | 0.16% | 0.60% | -0.15% | 0.54% | 0.29% | 0.70% | 0.57% | 0.78% | 0.53% | 0.51% | 0.63% | 0.40% | 0.59% | 0.44% | 0.47% |
| Factiss | -0.26% | 6.66% | 1.78% | 5.40% | 5.22% | -0.52% | 4.51% | -1.28% | 2.67% | -2.00% | -2.67% | 4.75% | -4.00% | 3.24% | -5.18% |
| Fzdinen | 2.18% | 0.84% | 0.06% | 1.30% | -0.88% | 2.32% | 1.03% | -0.82% | 0.83% | -1.31% | 1.88% | 1.08% | -2.55% | 0.57% | -2.56% |
| Frozen pizza | -0.49% | -0.26% | -0.88% | 0.07% | 0.05% | 0.57% | -0.02% | -1.75% | 0.11% | -1.24% | 0.86% | 0.03% | -2.15% | 0.07% | -1.75% |
| Household Cleaner | 2.26% | 1.08% | 6.20% | 0.98% | 4.04% | -0.12% | 1.32% | 3.18% | 1.64% | 2.72% | -1.43% | 2.10% | 0.14% | 2.22% | 0.20% |
| Hotdog | -1.13% | 0.12% | -6.92% | 0.77% | -4.00% | -0.16% | -0.65% | -5.99% | -0.13% | -4.14% | 0.45% | -0.52% | -6.36% | -0.20% | -4.86% |
| Laundry Detergent | 1.20% | 1.63% | 2.81% | 1.47% | 2.37% | 1.55% | 0.99% | 2.10% | 1.17% | 2.30% | 1.06% | 0.79% | -0.45% | 0.89% | 0.33% |
| Margarine/Butter | -4.16% | -0.42% | 2.22% | -0.82% | 2.69% | -1.78% | -0.60% | 1.01% | -0.72% | 1.97% | -1.21% | -0.82% | -1.15% | -0.99% | -1.03% |
| Mayonnaise | 1.31% | 1.11% | 7.88% | 0.63% | 7.94% | 0.18% | 0.04% | 1.64% | -0.12% | 0.91% | 0.40% | 0.26% | -2.49% | 0.14% | -2.64% |
| Milk | 1.26% | 1.90% | 6.89% | 1.86% | 5.80% | -0.15% | 1.72% | 4.24% | 1.37% | 3.28% | 0.12% | 2.23% | 5.57% | 2.15% | 4.65% |
| Mustard & Ketchup | -0.44% | 1.10% | 5.17% | 1.81% | 4.86% | -0.11% | 0.53% | 1.60% | 0.54% | 1.61% | 0.15% | 0.71% | 0.56% | 0.87% | 0.06% |
| Peanut butter | -0.64% | -2.62% | 3.55% | -3.19% | 3.28% | 0.06% | 1.07% | 5.42% | 0.94% | 4.87% | 0.25% | 0.17% | 4.43% | 0.11% | 3.77% |
| Photo | -0.10% | 1.24% | 7.19% | 1.14% | 6.60% | 0.05% | 1.49% | 4.39% | 1.10% | 4.62% | 0.11% | 1.17% | 0.90% | 1.02% | 0.87% |
| Razors | 0.00% | -1.08% | -3.37% | -1.08% | -3.37% | 0.00% | -1.74% | -2.24% | -1.74% | -2.24% | 0.00% | -1.59% | -4.91% | -1.59% | -4.91% |
| Salty snacks | 0.00% | 1.74% | 0.74% | 1.84% | 1.79% | 0.93% | 0.74% | -0.20% | 0.77% | 0.10% | 0.50% | 0.84% | 0.29% | 0.80% | 0.02% |
| Shamp | 1.46% | 0.84% | 2.82% | 0.69% | 2.67% | 0.64% | 0.88% | 1.60% | 0.79% | 1.25% | 1.07% | 0.86% | 1.39% | 0.70% | 1.10% |
| Soup | 2.80% | 1.47% | 4.21% | 1.19% | 3.90% | 2.45% | 1.22% | 1.29% | 0.99% | 1.44% | 2.05% | 1.29% | -1.70% | 0.98% | -1.25% |
| Spagsau | 2.15% | 4.98% | 4.46% | 3.14% | 2.79% | 1.04% | 5.19% | 1.19% | 4.37% | 0.29% | 1.47% | 4.65% | 1.66% | 4.13% | 0.75% |
| Sugar substitutes | 0.73% | 0.50% | 1.31% | 0.59% | 0.96% | 0.83% | 0.60% | 3.92% | 0.80% | 3.84% | 1.19% | 0.75% | 3.54% | 0.78% | 2.40% |
| Toilet Tissue | -0.06% | -0.44% | 9.13% | -1.23% | 9.35% | -0.72% | 0.20% | 5.37% | -0.30% | 6.09% | -1.26% | -0.74% | -0.05% | -0.52% | 0.80% |
| Toothbrush | 3.43% | -0.27% | -2.63% | -0.44% | -3.23% | 0.48% | -0.89% | -2.60% | -0.94% | -2.58% | 0.29% | -0.60% | -3.07% | -0.55% | -2.95% |
| Toothpaste | -0.67% | 0.10% | -1.83% | 1.09% | -2.09% | 3.21% | 0.26% | -0.82% | 0.77% | -0.90% | 3.14% | -0.13% | -0.17% | 0.37% | 0.30% |
| Yogurt | -0.91% | 2.92% | 4.59% | 2.72% | 3.75% | 0.08% | 3.13% | 4.77% | 2.68% | 3.95% | 0.23% | 3.36% | 4.06% | 2.98% | 2.81% |

Table 4d Comparing forecasting performance for each product category for the AvgRelMAE

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Category | Horizon=1 | | | | | | Horizon=4 | | | | | | Horizon=12 | | | | | |
| RelAvgMAE | | | Improvement based on RelAvgMAE | | RelAvgMAE | | | | Improvement based on RelAvgMAE | | RelAvgMAE | | | | Improvement based on RelAvgMAE | |
| ADL-own versus ADL-intra | ADL-own versus ADL-own-EWC | ADL-own versus ADL-own-IC | ADL-intra versus ADL-intra-EWC | ADL-intra versus ADL-intra-IC | ADL-own versus ADL-intra | | ADL-own versus ADL-own-EWC | ADL-own versus ADL-own-IC | ADL-intra versus ADL-intra-EWC | ADL-intra versus ADL-intra-IC | ADL-own versus ADL-intra | | ADL-own versus ADL-own-EWC | ADL-own versus ADL-own-IC | ADL-intra versus ADL-intra-EWC | ADL-intra versus ADL-intra-IC |
| Paptowl | 0.982 | 1.028 | 0.874 | -0.013 | 0.141 | 0.988 | | 0.948 | 1.113 | 0.056 | -0.114 | 0.995 | | 1.019 | 1.033 | -0.018 | -0.033 |
| Beer | 0.997 | 1.006 | 0.990 | 0.023 | 0.004 | 0.991 | | 0.992 | 0.993 | 0.008 | 0.008 | 0.993 | | 0.993 | 1.004 | 0.007 | -0.004 |
| Blades | 1.004 | 1.056 | 0.981 | -0.056 | 0.030 | 0.987 | | 0.991 | 0.968 | 0.004 | 0.021 | 0.992 | | 0.982 | 0.966 | 0.014 | 0.021 |
| Carbonated Beverages | 0.971 | 1.029 | 1.000 | -0.016 | -0.037 | 0.987 | | 1.017 | 1.024 | -0.016 | -0.029 | 0.992 | | 1.019 | 1.023 | -0.016 | -0.021 |
| Cigarette | 1.011 | 1.022 | 0.988 | -0.011 | 0.022 | 1.001 | | 0.989 | 0.991 | 0.012 | 0.007 | 1.002 | | 0.995 | 0.993 | 0.006 | 0.007 |
| Coffee | 1.014 | 0.998 | 0.930 | 0.005 | 0.054 | 0.993 | | 1.002 | 0.987 | -0.004 | 0.006 | 0.997 | | 1.005 | 0.993 | -0.003 | 0.005 |
| Coldcer | 0.984 | 1.031 | 1.034 | -0.031 | -0.036 | 1.002 | | 1.018 | 1.039 | -0.011 | -0.029 | 0.994 | | 1.009 | 1.025 | -0.004 | -0.018 |
| Deod | 0.999 | 0.968 | 0.999 | 0.019 | 0.009 | 0.992 | | 0.990 | 0.980 | 0.009 | 0.016 | 0.994 | | 0.993 | 0.983 | 0.007 | 0.015 |
| Factiss | 0.978 | 0.965 | 1.045 | 0.062 | -0.012 | 1.007 | | 0.956 | 1.002 | 0.026 | 0.001 | 1.003 | | 0.950 | 1.028 | 0.033 | -0.037 |
| Fzdinen | 1.048 | 0.985 | 1.054 | 0.005 | 0.017 | 0.972 | | 0.989 | 1.038 | 0.016 | -0.028 | 0.977 | | 0.990 | 1.043 | 0.003 | -0.035 |
| Frozen pizza | 1.009 | 1.006 | 0.994 | 0.025 | 0.021 | 0.995 | | 1.005 | 1.033 | -0.003 | -0.029 | 0.988 | | 0.999 | 1.039 | 0.001 | -0.035 |
| Household Cleaner | 0.930 | 0.983 | 0.878 | 0.002 | 0.029 | 1.000 | | 0.979 | 0.955 | 0.026 | 0.038 | 1.011 | | 0.978 | 0.985 | 0.024 | 0.015 |
| Hotdog | 1.101 | 1.017 | 1.199 | 0.024 | -0.088 | 1.004 | | 1.008 | 1.092 | -0.002 | -0.069 | 0.985 | | 1.004 | 1.079 | -0.002 | -0.071 |
| Laundry Detergent | 0.988 | 0.971 | 0.929 | 0.031 | 0.064 | 0.980 | | 0.997 | 0.965 | 0.005 | 0.040 | 0.991 | | 0.991 | 1.010 | 0.011 | 0.008 |
| Margarine/Butter | 1.064 | 1.068 | 1.060 | -0.048 | -0.030 | 1.025 | | 1.024 | 1.009 | -0.019 | 0.009 | 1.013 | | 1.023 | 1.016 | -0.021 | -0.016 |
| Mayonnaise | 0.931 | 0.956 | 0.881 | 0.052 | 0.121 | 0.999 | | 0.997 | 0.990 | 0.000 | 0.003 | 0.995 | | 0.996 | 1.026 | 0.002 | -0.033 |
| Milk | 0.953 | 0.966 | 0.921 | 0.046 | 0.043 | 0.992 | | 0.978 | 0.935 | 0.018 | 0.044 | 0.995 | | 0.979 | 0.943 | 0.019 | 0.043 |
| Mustard & Ketchup | 1.110 | 1.067 | 0.955 | 0.013 | 0.141 | 1.012 | | 0.995 | 0.984 | 0.008 | 0.023 | 1.000 | | 0.988 | 0.993 | 0.014 | 0.003 |
| Peanut butter | 1.067 | 1.042 | 1.040 | -0.042 | 0.016 | 1.009 | | 0.982 | 0.979 | 0.016 | 0.025 | 0.995 | | 1.003 | 0.971 | -0.004 | 0.021 |
| Photo | 1.028 | 0.967 | 0.918 | 0.048 | 0.085 | 0.998 | | 0.981 | 0.946 | 0.015 | 0.058 | 0.997 | | 0.982 | 0.990 | 0.016 | 0.012 |
| Razors | 1.000 | 0.918 | 1.040 | 0.082 | -0.040 | 1.000 | | 1.034 | 1.003 | -0.034 | -0.003 | 1.000 | | 1.024 | 1.048 | -0.024 | -0.048 |
| Salty snacks | 0.992 | 0.970 | 1.029 | 0.045 | -0.018 | 0.977 | | 0.988 | 0.999 | 0.011 | -0.005 | 0.988 | | 0.995 | 0.991 | 0.005 | 0.000 |
| Shamp | 0.979 | 0.980 | 0.858 | 0.013 | 0.103 | 0.995 | | 0.989 | 0.970 | 0.011 | 0.023 | 0.990 | | 0.988 | 0.978 | 0.010 | 0.013 |
| Soup | 0.960 | 0.975 | 0.891 | 0.037 | 0.088 | 0.964 | | 0.986 | 0.976 | 0.010 | 0.016 | 0.977 | | 0.988 | 1.013 | 0.012 | -0.009 |
| Spagsau | 0.995 | 0.962 | 0.964 | 0.027 | 0.030 | 0.990 | | 0.915 | 0.975 | 0.071 | 0.015 | 0.994 | | 0.925 | 0.977 | 0.071 | 0.016 |
| Sugar substitutes | 0.990 | 0.973 | 0.973 | 0.049 | 0.005 | 0.986 | | 0.995 | 0.955 | 0.009 | 0.042 | 0.984 | | 0.992 | 0.953 | 0.008 | 0.030 |
| Toilet Tissue | 0.976 | 0.983 | 0.827 | -0.030 | 0.129 | 0.991 | | 1.002 | 0.921 | -0.025 | 0.069 | 0.997 | | 0.995 | 0.981 | 0.000 | 0.017 |
| Toothbrush | 0.932 | 1.037 | 1.007 | -0.040 | -0.005 | 0.993 | | 1.016 | 1.049 | -0.015 | -0.044 | 0.993 | | 1.009 | 1.040 | -0.007 | -0.040 |
| Toothpaste | 0.986 | 0.981 | 1.023 | 0.060 | 0.008 | 0.981 | | 1.002 | 1.061 | 0.011 | -0.030 | 1.025 | | 1.007 | 1.027 | 0.005 | 0.012 |
| Yogurt | 1.014 | 0.955 | 0.978 | 0.028 | 0.038 | 1.000 | | 0.967 | 0.951 | 0.033 | 0.050 | 0.999 | | 0.964 | 0.953 | 0.034 | 0.035 |

8.4 Explore the determinants of the forecasting improvement

The results in section 8.4 shows that our proposed models generate more accurate forecasts for most of the product categories. In this section, we further explore the determinants of the improvement of the forecasting performance. We take the percentage improvement of the forecasting accuracy as the dependent variable, and we include the following explanatory variables 1) basic statistical measures for both prices and sales including the average, standard deviation, skewness, range, kurtosis, and coefficient of variation; 2) the percentage of feature and display promotional time periods for each product. 3) Three measures to capture the characteristics of the data series following Fildes et al. (1998). For example, we measure the proportion of outliers for the sales of each product. The value of the sales for product *i* will be identified as an outlier if or , where is the differenced value of the sales for product *i*. and are the first and third quantiles of . This measure may suggest the difficulty to generate accurate sales forecasts for this product. We also measure the randomness by regressing on , where is the sales value for product *i* at week *t* and *t* is the time trend. The fitness of this autoregressive model (e.g., the R square) may approximate the systematic variation in the sales data series which may be captured by simple models. Lastly, we measure the linear trend for the sales of each product as the absolute correlation between and the time trend. 4) dummy variables for each of the product category. Table 6 exhibits the explanatory variables.

Table 5. the explanatory variables (excluding category dummies) we use to explore the determinants of forecasting performance improvement.

|  |  |
| --- | --- |
| Variables | Descriptions |
| Price\_mean | Average price |
| Price\_std | Standard deviation of the unit sales |
| Price\_SKEWNESS | Sknewness of the price |
| Price\_range | Range of the price |
| Price\_kurtosis | Kurtosis of the price |
| Price\_coefficient of variation | Coefficient of variation of the price |
| Sales\_mean | Average sales |
| Sales\_std | Standard deviation of the unit sales |
| Sales\_SKEWNESS | Sknewness of the sales |
| Sales\_range | Range of the sales |
| Sales\_kurtosis | Kurtosis of the sales |
| Sales\_coefficient of variation | Coefficient of variation of the sales |
| D\_freq | Percentage of display |
| F\_freq | Percentage of feature |
| Outliers\_pct | Percentage of outliers |
| Randomness | Measure of randomness |
| Linear\_trend | Measure of linear trend |

We construct the following model:

where represents the percentage of improvement by model *A* over model *B* for product *i*. We assume that . Table 7 exhibits the estimates of the parameters for four pair of models based on error measures including the MAPE, the SMAPE, and the MASE. For example, column two to column are for ‘ADL- intra- IC versus ADL-Intra’ representing the model with the dependant variable for the various error measures. The results of the twelve models indicate: 1) the models tend to have better forecasting performance for the product with higher price for moderate and long forecast horizons (e.g., when h=4 and 12); 2) the models tend to have better forecasting performance for the product with low price variations, low sales variations, or low coefficients of variations for sales. One explanation is that the improved forecasting accuracy could be immerged in the variations in sales (which are to some extend caused by the variations in prices). 3) the models tend to have better forecasting performance for the product with less frequent display advertising. However, the evidence is significant for long forecast horizon (e.g., h=12) but insignificant for moderate and short horizons. 4) The models tend to have better forecasting performance for the product with low randomness. This also indicates that the improved forecasting accuracy could be immerged in the variations in the sales of the product with high randomness. 5) The models tend to have better forecasting performance for the product with high linear trend, especially when forecast horizons get longer (e.g., h=4 and 12). The trend may indicate that there is a change in product sales due to the changing effectiveness of the marketing activities which however cannot be capture by the time trend variable.

Table 6a. The estimation results for the response model across 1 to 12-week-ahead forecast horizons

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Parameters/ h=12 | ADL- intra- IC versus ADL-Intra | | | | ADL- intra- EWC versus ADL-Intra | | | ADL- OWN- IC versus ADL-OWN | | | ADL- OWN- EWC versus ADL-OWN | | | |
| **MAPE** | **SMAPE** | **MASE** | **MAPE** | | **SMAPE** | **MASE** | **MAPE** | **SMAPE** | **MASE** | **MAPE** | **SMAPE** | **MASE** |
| Intercept | -0.030 | -0.004 | -0.010 | 0.014 | | 0.019\*\*\* | 0.019\*\* | -0.027 | -0.002 | -0.008 | 0.013 | 0.017\*\* | 0.017\* |
| price\_mean | 0.004\*\*\* | 0.001\* | 0.002\*\*\* | 0.001\*\*\* | | 0.001\*\* | 0.001\*\* | 0.005\*\*\* | 0.002\*\*\* | 0.003\*\*\* | 0.001\*\* | 0.001\*\* | 0.001\*\*\* |
| price\_std | -0.045\*\* | -0.03\*\* | -0.042\*\* | -0.015 | | -0.014\*\* | -0.014\* | -0.047\*\* | -0.035\*\*\* | -0.049\*\*\* | -0.018 | -0.019\*\* | -0.022\*\* |
| price\_skewness | -0.003 | -0.002 | -0.003 | 0.000 | | 0.000 | 0.000 | -0.002 | -0.002 | -0.003 | -0.002 | -0.001 | -0.001 |
| price\_range | 0.001 | 0.006\*\* | 0.005 | -0.002 | | 0.001 | 0.001 | -0.001 | 0.005 | 0.003 | -0.002 | 0.002 | 0.001 |
| price\_kurtosis | -0.001\*\* | 0.000\* | 0.000 | 0.000\*\* | | 0.000\* | 0.000\*\* | -0.001\*\*\* | 0.000\*\* | -0.001\*\* | 0.000\*\* | 0.000\* | 0.000\* |
| price\_c\_v | 0.197 | 0.023 | 0.065 | 0.053 | | 0.010 | 0.010 | 0.179 | 0.045 | 0.092 | 0.065 | 0.058 | 0.062 |
| sales\_mean | 0.000\*\*\* | 0.000\*\* | 0.000\*\* | 0.000 | | 0.000 | 0.000 | 0.000\*\*\* | 0.000\*\* | 0.000\*\* | 0.000 | 0.000 | 0.000 |
| sales\_std | -0.001\*\*\* | 0.000\*\* | -0.001\*\* | 0.000\* | | 0.000 | 0.000 | -0.001\*\*\* | -0.001\*\* | -0.001\*\* | 0.000\*\* | 0.000\*\* | 0.000\* |
| sales\_skewness | 0.013 | 0.007 | 0.013 | 0.000 | | -0.001 | -0.001 | 0.000 | -0.001 | 0.002 | 0.001 | 0.001 | 0.001 |
| sales\_range | 0.000\*\*\* | 0.000\*\* | 0.000\*\* | 0.000\*\* | | 0.000\* | 0.000\* | 0.000\*\*\* | 0.000\*\* | 0.000\*\* | 0.000\*\* | 0.000\*\* | 0.000\* |
| sales\_kurtosis | -0.002\* | -0.001 | -0.002\* | 0.000 | | 0.000 | 0.000 | -0.001 | 0.000 | -0.001 | 0.000 | 0.000 | 0.000 |
| sales\_c\_v | -0.038\*\* | -0.012 | -0.021 | -0.007 | | -0.001 | -0.001 | -0.038\* | -0.006 | -0.020 | 0.000 | 0.003 | 0.003 |
| d\_freq | -0.092\*\* | -0.063\*\* | -0.091\*\* | -0.025 | | -0.019 | -0.019 | -0.079\* | -0.052\* | -0.093\*\* | -0.046\*\* | -0.038\*\* | -0.049\*\* |
| f\_freq | 0.047 | 0.001 | 0.033 | 0.023 | | -0.007 | -0.007 | 0.043 | 0.000 | 0.041 | -0.004 | -0.017 | -0.031 |
| outliers\_pct | -0.130 | -0.049 | -0.056 | -0.020 | | 0.006 | 0.006 | -0.033 | -0.016 | 0.026 | -0.046 | -0.018 | -0.004 |
| randomness | -0.021 | 0.023 | 0.014 | 0.021\* | | 0.027\*\*\* | 0.027\*\*\* | -0.003 | 0.038\*\* | 0.034 | 0.021\* | 0.022\*\*\* | 0.024\*\* |
| linear\_trend | 0.117\*\*\* | 0.043\*\*\* | 0.059\*\*\* | 0.036\*\*\* | | 0.019\*\*\* | 0.019\*\*\* | 0.124\*\*\* | 0.045\*\*\* | 0.063\*\*\* | 0.031\*\*\* | 0.017\*\*\* | 0.025\*\*\* |
| category\_beer | 0.036 | 0.003 | 0.009 | -0.001 | | -0.012\* | -0.012 | 0.026 | -0.003 | 0.003 | 0.002 | -0.010 | -0.007 |
| category\_blades | 0.041 | -0.002 | 0.009 | 0.004 | | -0.013 | -0.013 | 0.047 | 0.002 | 0.017 | 0.007 | -0.011 | -0.006 |
| category\_carbbev | -0.031 | -0.024 | -0.047\* | -0.057\*\*\* | | -0.039\*\*\* | -0.039\*\*\* | -0.030 | -0.024 | -0.044\* | -0.06\*\*\* | -0.039\*\*\* | -0.056\*\*\* |
| category\_cigets | -0.033 | -0.010 | -0.015 | -0.021\* | | -0.02\*\* | -0.02\*\* | -0.040 | -0.015 | -0.019 | -0.017 | -0.022\*\* | -0.022\*\* |
| category\_coffee | 0.039 | 0.000 | 0.008 | -0.028\*\*\* | | -0.027\*\*\* | -0.027\*\*\* | 0.046\* | 0.000 | 0.007 | -0.03\*\*\* | -0.03\*\*\* | -0.032\*\*\* |
| category\_coldcer | 0.000 | -0.015 | -0.022 | -0.027\*\* | | -0.024\*\*\* | -0.024\*\* | 0.009 | -0.024 | -0.025 | -0.029\*\* | -0.03\*\*\* | -0.03\*\*\* |
| category\_deod | 0.068\*\*\* | 0.009 | 0.018 | -0.001 | | -0.013\* | -0.013 | 0.074\*\*\* | 0.010 | 0.020 | -0.004 | -0.016\*\* | -0.015\* |
| category\_factiss | -0.051 | -0.063\*\* | -0.094\* | 0.023 | | -0.003 | -0.003 | -0.047 | -0.065\*\* | -0.099\*\* | 0.027 | 0.001 | 0.004 |
| category\_fzdinen | 0.105\*\*\* | -0.007 | 0.005 | 0.011 | | -0.013 | -0.013 | 0.11\*\*\* | -0.011 | 0.002 | 0.021\* | -0.010 | -0.006 |
| category\_fzpizza | -0.017 | -0.014 | -0.019 | -0.03\*\*\* | | -0.02\*\*\* | -0.02\*\*\* | -0.020 | -0.021 | -0.026 | -0.023\*\* | -0.021\*\*\* | -0.025\*\*\* |
| category\_hhclean | -0.010 | -0.012 | -0.015 | 0.021 | | 0.006 | 0.006 | -0.004 | -0.015 | -0.017 | 0.023 | 0.004 | 0.013 |
| category\_hotdog | -0.108\*\* | -0.072\*\*\* | -0.109\*\*\* | -0.031\*\* | | -0.026\*\*\* | -0.026\*\* | -0.113\*\* | -0.097\*\*\* | -0.143\*\*\* | -0.033\*\* | -0.035\*\*\* | -0.037\*\*\* |
| category\_laundet | 0.023 | -0.001 | 0.002 | 0.014 | | -0.012 | -0.012 | 0.008 | -0.008 | -0.011 | 0.014 | -0.015\* | -0.006 |
| category\_margbut | 0.001 | -0.020 | -0.026 | -0.027\*\* | | -0.029\*\*\* | -0.029\*\*\* | 0.006 | -0.022 | -0.027 | -0.025\*\* | -0.029\*\*\* | -0.03\*\*\* |
| category\_mayo | -0.016 | -0.046\*\* | -0.052\* | -0.008 | | -0.026\*\*\* | -0.026\*\* | -0.012 | -0.05\* | -0.051 | -0.004 | -0.024\*\* | -0.023\*\* |
| category\_milk | 0.008 | 0.002 | 0.001 | -0.011 | | -0.012 | -0.012 | 0.002 | 0.000 | -0.003 | -0.009 | -0.014 | -0.016 |
| category\_mustket | 0.010 | -0.016 | -0.018 | 0.000 | | -0.02\*\* | -0.020 | 0.024 | -0.016 | -0.015 | 0.000 | -0.024\*\*\* | -0.021\* |
| category\_Paptowl | 0.184\* | 0.065 | 0.192\*\* | -0.059 | | -0.06\*\*\* | -0.06\* | 0.201\*\* | 0.059 | 0.193\*\* | -0.065 | -0.067\*\*\* | -0.086\* |
| category\_peanbut | 0.018 | 0.022 | 0.023 | 0.005 | | -0.032\*\* | -0.032\*\* | 0.021 | 0.024 | 0.025 | 0.004 | -0.033\*\* | -0.031\*\* |
| category\_photo | 0.113\*\*\* | 0.022 | 0.042 | 0.008 | | -0.006 | -0.006 | 0.115\*\*\* | 0.020 | 0.043 | 0.013 | -0.005 | 0.004 |
| category\_razors | 0.090 | -0.031 | -0.004 | -0.041 | | -0.043\*\* | -0.043\* | 0.092 | -0.038 | -0.005 | -0.040 | -0.041\*\* | -0.043\*\* |
| category\_saltsnc | 0.029 | 0.007 | 0.011 | -0.001 | | -0.009 | -0.009 | 0.028 | 0.008 | 0.010 | -0.002 | -0.010 | -0.009 |
| category\_shamp | 0.059\* | 0.020 | 0.022 | 0.010 | | -0.008 | -0.008 | 0.077\*\* | 0.019 | 0.026 | 0.012 | -0.010 | -0.004 |
| category\_soup | 0.036 | -0.011 | -0.026 | 0.003 | | -0.010 | -0.010 | 0.042 | -0.018 | -0.032 | 0.006 | -0.010 | -0.004 |
| category\_spagsau | 0.048\* | -0.011 | -0.005 | 0.05\*\*\* | | 0.008 | 0.008 | 0.062\*\* | -0.002 | 0.005 | 0.054\*\*\* | 0.011 | 0.022 |
| category\_sugarsu | 0.062\* | 0.033 | 0.036 | -0.005 | | -0.012 | -0.012 | 0.082\*\* | 0.042 | 0.052 | -0.008 | -0.016\*\* | -0.015\* |
| category\_toitisu | 0.071 | -0.004 | 0.006 | -0.006 | | -0.037\*\*\* | -0.037\* | 0.070 | -0.014 | -0.005 | -0.006 | -0.043\*\*\* | -0.048\*\* |
| category\_toothbr | -0.003 | -0.024 | -0.034 | -0.031\*\*\* | | -0.025\*\*\* | -0.025\*\*\* | -0.001 | -0.028 | -0.039 | -0.037\*\*\* | -0.028\*\*\* | -0.033\*\*\* |
| category\_toothpa | 0.08\* | 0.035 | 0.058 | -0.009 | | -0.017\*\* | -0.017 | 0.067 | 0.026 | 0.044 | -0.014 | -0.02\*\* | -0.019 |

Table 6b. The estimation results for the response model across 1 to 4-week-ahead forecast horizons

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Parameters | ADL- intra- IC versus ADL-Intra | | | ADL- intra- EWC versus ADL-Intra | | | ADL- OWN- IC versus ADL-OWN | | | ADL- OWN- EWC versus ADL-OWN | | |
| **MAPE** | **SMAPE** | **MASE** | **MAPE** | **SMAPE** | **MASE** | **MAPE** | **SMAPE** | **MASE** | **MAPE** | **SMAPE** | **MASE** |
| Intercept | -0.023 | 0.002 | -0.005 | 0.005 | 0.012 | 0.012 | -0.036 | -0.010 | -0.019 | 0.005 | 0.010 | 0.010 |
| price\_mean | 0.006\*\*\* | 0.003\*\*\* | 0.004\*\*\* | 0.001\* | 0.000 | 0.000 | 0.007\*\*\* | 0.004\*\*\* | 0.005\*\*\* | 0.001\*\* | 0.001\* | 0.001\*\* |
| price\_std | -0.054\*\* | -0.042\*\* | -0.055\*\* | -0.017\* | -0.015\*\* | -0.015\*\* | -0.062\*\* | -0.052\*\*\* | -0.068\*\*\* | -0.019\* | -0.018\*\* | -0.022\*\* |
| price\_skewness | 0.004 | 0.003 | 0.002 | 0.000 | 0.000 | 0.000 | 0.004 | 0.003 | 0.002 | -0.002 | -0.001 | -0.001 |
| price\_range | -0.002 | 0.004 | 0.003 | 0.000 | 0.002 | 0.002 | -0.005 | 0.002 | 0.001 | -0.001 | 0.002 | 0.001 |
| price\_kurtosis | 0.000 | 0.000 | 0.000 | 0.000\*\* | 0.000\*\* | 0.000\*\* | 0.000 | 0.000 | 0.000 | 0.000\*\* | 0.000\*\* | 0.000\*\*\* |
| price\_c\_v | 0.325\* | 0.138 | 0.213 | 0.074 | 0.020 | 0.020 | 0.379\*\* | 0.217\*\* | 0.315\*\* | 0.103\* | 0.067\* | 0.076 |
| sales\_mean | 0.000\*\*\* | 0.000\*\*\* | 0.000\*\*\* | 0.000 | 0.000 | 0.000 | 0.000\*\*\* | 0.000\*\*\* | 0.000\*\*\* | 0.000\* | 0.000 | 0.000 |
| sales\_std | -0.002\*\*\* | -0.001\*\* | -0.001\*\* | 0.000\* | 0.000 | 0.000\* | -0.002\*\*\* | -0.001\*\* | -0.002\*\* | 0.000\* | 0.000 | 0.000\* |
| sales\_skewness | 0.020 | 0.009 | 0.022\*\* | 0.001 | -0.001 | -0.001 | 0.009 | 0.005 | 0.014 | 0.001 | 0.000 | 0.002 |
| sales\_range | 0.000\*\*\* | 0.000\*\* | 0.000\*\* | 0.000 | 0.000 | 0.000 | 0.000\*\*\* | 0.000\*\* | 0.000\*\* | 0.000\* | 0.000 | 0.000\* |
| sales\_kurtosis | -0.003\* | -0.001 | -0.003\* | 0.000 | 0.000 | 0.000 | -0.002 | 0.000 | -0.002 | 0.000 | 0.000 | 0.000 |
| sales\_c\_v | -0.067\*\*\* | -0.03\*\* | -0.046\*\* | -0.005 | 0.002 | 0.002 | -0.074\*\*\* | -0.029\*\* | -0.048\*\* | 0.001 | 0.004 | 0.002 |
| d\_freq | -0.040 | -0.005 | -0.030 | -0.014 | -0.011 | -0.011 | -0.019 | 0.006 | -0.030 | -0.022 | -0.016 | -0.011 |
| f\_freq | 0.028 | 0.006 | 0.058 | 0.044 | 0.002 | 0.002 | 0.034 | 0.024 | 0.096 | 0.010 | -0.005 | -0.028 |
| outliers\_pct | -0.081 | -0.042 | -0.096 | -0.022 | 0.001 | 0.001 | 0.022 | -0.027 | -0.054 | -0.048 | -0.018 | -0.006 |
| randomness | -0.042 | 0.003 | -0.012 | 0.021\* | 0.028\*\*\* | 0.028\*\*\* | -0.005 | 0.029 | 0.023 | 0.019\* | 0.024\*\*\* | 0.026\*\* |
| linear\_trend | 0.112\*\*\* | 0.037\*\*\* | 0.056\*\*\* | 0.033\*\*\* | 0.016\*\*\* | 0.016\*\*\* | 0.116\*\*\* | 0.042\*\*\* | 0.066\*\*\* | 0.032\*\*\* | 0.016\*\*\* | 0.023\*\*\* |
| category\_beer | 0.039 | 0.002 | 0.007 | 0.005 | -0.008 | -0.008 | 0.039 | 0.007 | 0.013 | 0.007 | -0.007 | -0.005 |
| category\_blades | 0.029 | -0.011 | 0.001 | 0.004 | -0.014 | -0.014 | 0.041 | -0.003 | 0.015 | 0.006 | -0.012 | -0.009 |
| category\_carbbev | -0.026 | -0.025 | -0.048\* | -0.05\*\*\* | -0.032\*\*\* | -0.032\*\*\* | -0.017 | -0.020 | -0.046 | -0.051\*\*\* | -0.033\*\*\* | -0.045\*\*\* |
| category\_cigets | -0.052 | -0.028 | -0.031 | -0.003 | -0.009 | -0.009 | -0.045 | -0.021 | -0.022 | -0.005 | -0.011 | -0.010 |
| category\_coffee | 0.031 | -0.003 | 0.007 | -0.022\*\* | -0.02\*\* | -0.02\*\* | 0.048 | 0.005 | 0.015 | -0.024\*\* | -0.025\*\*\* | -0.025\*\* |
| category\_coldcer | 0.025 | 0.001 | 0.001 | -0.032\*\* | -0.024\*\* | -0.024\*\* | 0.034 | 0.001 | 0.010 | -0.027\*\* | -0.028\*\*\* | -0.026\*\* |
| category\_deod | 0.064\*\*\* | 0.005 | 0.010 | 0.004 | -0.007 | -0.007 | 0.079\*\*\* | 0.014 | 0.023 | 0.002 | -0.010 | -0.010 |
| category\_factiss | -0.037 | -0.046\* | -0.066\* | 0.002 | -0.006 | -0.006 | -0.031 | -0.041 | -0.067\* | 0.004 | -0.003 | -0.002 |
| category\_fzdinen | 0.135\*\*\* | 0.007 | 0.020 | 0.013 | -0.007 | -0.007 | 0.15\*\*\* | 0.013 | 0.034 | 0.018 | -0.010 | -0.004 |
| category\_fzpizza | -0.025 | -0.015 | -0.019 | -0.028\*\* | -0.015\* | -0.015\*\* | -0.020 | -0.015 | -0.019 | -0.019\*\* | -0.017\*\* | -0.023\*\* |
| category\_hhclean | -0.005 | -0.007 | -0.010 | 0.022 | 0.005 | 0.005 | 0.008 | -0.007 | -0.007 | 0.020 | 0.000 | 0.006 |
| category\_hotdog | -0.123\*\* | -0.084\*\* | -0.135\*\* | -0.03\* | -0.027\*\* | -0.027\*\* | -0.136\*\* | -0.106\*\*\* | -0.158\*\*\* | -0.04\*\* | -0.042\*\*\* | -0.05\*\*\* |
| category\_laundet | 0.071\*\* | 0.024 | 0.040 | 0.023\* | -0.002 | -0.002 | 0.071\*\* | 0.033 | 0.042 | 0.023\* | -0.005 | 0.006 |
| category\_margbut | 0.029 | 0.000 | -0.008 | -0.019\* | -0.018\*\* | -0.018\*\* | 0.024 | -0.011 | -0.020 | -0.017 | -0.021\*\* | -0.024\*\* |
| category\_mayo | 0.025 | -0.013 | -0.009 | -0.006 | -0.023\*\* | -0.023\*\* | 0.038 | -0.008 | 0.002 | -0.003 | -0.022\*\* | -0.021\*\* |
| category\_milk | -0.011 | -0.015 | -0.014 | -0.008 | -0.011 | -0.011 | -0.012 | -0.010 | -0.009 | -0.006 | -0.013 | -0.014 |
| category\_mustket | 0.018 | -0.010 | -0.009 | 0.005 | -0.018\* | -0.018 | 0.028 | -0.015 | -0.017 | 0.007 | -0.021\*\* | -0.018 |
| category\_Paptowl | 0.22\*\*\* | 0.067 | 0.218\*\*\* | -0.027 | -0.037 | -0.037 | 0.253\*\*\* | 0.082 | 0.236\*\*\* | -0.029 | -0.044 | -0.062 |
| category\_peanbut | -0.009 | 0.004 | 0.007 | 0.020 | -0.007 | -0.007 | 0.000 | 0.013 | 0.018 | 0.020 | -0.008 | -0.006 |
| category\_photo | 0.145\*\*\* | 0.057\* | 0.078\*\* | 0.014 | -0.001 | -0.001 | 0.149\*\*\* | 0.061\*\* | 0.084\*\* | 0.019 | 0.000 | 0.012 |
| category\_razors | 0.14\*\*\* | -0.007 | 0.034 | -0.031\* | -0.037\*\*\* | -0.037\*\* | 0.146\*\*\* | -0.005 | 0.041 | -0.031\*\* | -0.037\*\*\* | -0.041\*\*\* |
| category\_saltsnc | 0.027 | -0.004 | 0.005 | 0.001 | -0.004 | -0.004 | 0.023 | -0.002 | 0.009 | 0.003 | -0.006 | -0.005 |
| category\_shamp | 0.049 | 0.009 | 0.007 | 0.009 | -0.004 | -0.004 | 0.08\*\* | 0.023 | 0.029 | 0.011 | -0.007 | -0.003 |
| category\_soup | 0.069\*\*\* | 0.016 | -0.010 | 0.006 | -0.002 | -0.002 | 0.092\*\*\* | 0.019 | -0.007 | 0.010 | -0.003 | 0.003 |
| category\_spagsau | 0.063\*\* | -0.004 | 0.003 | 0.055\*\*\* | 0.017 | 0.017 | 0.084\*\* | 0.008 | 0.020 | 0.06\*\*\* | 0.021 | 0.032\* |
| category\_sugarsu | 0.062 | 0.035 | 0.038 | 0.001 | -0.005 | -0.005 | 0.084\*\* | 0.044 | 0.050 | -0.005 | -0.012 | -0.013 |
| category\_toitisu | 0.104\* | 0.06\* | 0.075 | -0.009 | -0.022\* | -0.022 | 0.11\* | 0.059\* | 0.073 | -0.002 | -0.024\*\* | -0.026 |
| category\_toothbr | -0.029 | -0.035 | -0.055 | -0.032\*\*\* | -0.025\*\*\* | -0.025\*\*\* | -0.020 | -0.033 | -0.055 | -0.036\*\*\* | -0.027\*\*\* | -0.033\*\*\* |
| category\_toothpa | 0.065 | 0.034 | 0.043 | -0.002 | -0.012 | -0.012 | 0.064 | 0.043 | 0.056 | -0.006 | -0.015 | -0.013 |

Table 6c. The estimation results for the response model for 1-week-ahead forecast horizon

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Parameters/ h=1 | ADL- intra- IC versus ADL-Intra | | | ADL- intra- EWC versus ADL-Intra | | | | ADL- OWN- IC versus ADL-OWN | | | | ADL- OWN- EWC versus ADL-OWN | | | |
| **MAPE** | **SMAPE** | **MASE** | **MAPE** | **SMAPE** | **MASE** | **MAPE** | | **SMAPE** | **MASE** | **MAPE** | | **SMAPE** | **MASE** |
| Intercept | -0.025 | 0.002 | 0.005 | 0.034 | 0.038\* | 0.038\* | -0.042 | | -0.024 | -0.036 | 0.012 | | 0.016 | 0.013 |
| price\_mean | 0.002 | -0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.003 | | 0.000 | 0.002 | 0.001 | | 0.000 | 0.000 |
| price\_std | -0.030 | -0.020 | -0.015 | -0.035\*\* | -0.032\*\* | -0.032\*\* | -0.016 | | -0.019 | -0.014 | -0.035\*\* | | -0.036\*\* | -0.04\*\* |
| price\_skewness | 0.01\* | 0.006 | 0.003 | -0.002 | -0.001 | -0.001 | 0.011\* | | 0.007 | 0.003 | -0.004\*\* | | -0.002 | -0.003 |
| price\_range | 0.001 | 0.006 | 0.002 | 0.006 | 0.008\* | 0.008\*\* | -0.004 | | 0.003 | -0.001 | 0.006 | | 0.008\* | 0.008\* |
| price\_kurtosis | 0.000 | 0.000 | -0.001 | -0.001\*\*\* | 0.000\*\* | 0.000\*\* | 0.000 | | 0.000 | -0.001 | -0.001\*\*\* | | 0.000\*\* | 0.000\* |
| price\_c\_v | 0.068 | -0.167 | -0.129 | 0.011 | -0.027 | -0.027 | 0.213 | | 0.081 | 0.222 | 0.150 | | 0.129 | 0.167\* |
| sales\_mean | 0.000\*\* | 0.000 | 0.000 | 0.000 | 0.000\* | 0.000 | 0.000\*\* | | 0.000\* | 0.000\* | 0.000 | | 0.000 | 0.000 |
| sales\_std | -0.001 | 0.000 | -0.001 | 0.000 | 0.000 | 0.000 | -0.001 | | -0.001 | -0.001 | 0.000 | | 0.000 | 0.000 |
| sales\_skewness | 0.033 | 0.010 | 0.034 | -0.003 | -0.004 | -0.004 | 0.026 | | 0.008 | 0.029 | -0.003 | | -0.005 | -0.003 |
| sales\_range | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000\* | | 0.000 | 0.000 | 0.000 | | 0.000 | 0.000 |
| sales\_kurtosis | -0.003 | 0.000 | -0.003 | 0.000 | 0.001 | 0.001 | -0.003 | | 0.000 | -0.003 | 0.000 | | 0.000 | 0.000 |
| sales\_c\_v | -0.083\*\* | -0.065\* | -0.097\* | -0.016 | -0.014 | -0.014 | -0.076\*\* | | -0.053\*\* | -0.068\*\* | 0.016 | | 0.014 | 0.016 |
| d\_freq | 0.027 | 0.077 | 0.106 | 0.125 | 0.115 | 0.115 | 0.007 | | 0.051 | 0.013 | 0.032 | | 0.022 | 0.009 |
| f\_freq | 0.173 | 0.118 | 0.129 | 0.038 | 0.001 | 0.001 | 0.020 | | -0.012 | 0.100 | 0.001 | | -0.014 | 0.020 |
| outliers\_pct | -0.414\*\* | -0.266\* | -0.475\*\* | -0.139\*\* | -0.1\* | -0.1\*\* | -0.406\* | | -0.309\* | -0.595\*\*\* | -0.197\*\*\* | | -0.133\*\* | -0.203\*\*\* |
| randomness | -0.057 | -0.011 | -0.030 | 0.067\*\*\* | 0.067\*\*\* | 0.067\*\* | -0.048 | | -0.010 | -0.034 | 0.026 | | 0.029 | 0.024 |
| linear\_trend | 0.119 | 0.045\*\* | 0.061\*\* | 0.019\*\* | 0.003 | 0.003 | 0.113 | | 0.038 | 0.059\* | 0.016 | | 0.001 | 0.005 |
| category\_beer | 0.076 | 0.051 | 0.042 | -0.022 | -0.032\* | -0.032 | 0.073 | | 0.064 | 0.063 | -0.007 | | -0.016 | -0.009 |
| category\_blades | 0.050 | 0.022 | 0.035 | -0.041\* | -0.053\*\* | -0.053\* | 0.062 | | 0.036 | 0.059 | -0.023 | | -0.035 | -0.026 |
| category\_carbbev | -0.086 | -0.080 | -0.126 | -0.15\*\* | -0.132\*\* | -0.132\* | -0.036 | | -0.022 | -0.084 | -0.071\*\*\* | | -0.056\*\* | -0.073\*\*\* |
| category\_cigets | 0.002 | 0.037 | 0.031 | -0.028 | -0.036\* | -0.036 | -0.008 | | 0.035 | 0.022 | -0.036\* | | -0.04\*\* | -0.038\*\* |
| category\_coffee | 0.104\* | 0.083\* | 0.096\* | -0.034\* | -0.035\* | -0.035\* | 0.108\* | | 0.083 | 0.099 | -0.041\*\* | | -0.04\*\* | -0.039\*\* |
| category\_coldcer | 0.12\* | 0.104\* | 0.112\* | -0.026 | -0.018 | -0.018 | 0.139\*\* | | 0.117\* | 0.106 | -0.04\* | | -0.038\*\* | -0.04\* |
| category\_deod | 0.1\*\* | 0.052 | 0.053 | -0.011 | -0.020 | -0.020 | 0.106\* | | 0.057 | 0.054 | -0.011 | | -0.018 | -0.017 |
| category\_factiss | 0.041 | 0.032 | 0.014 | 0.010 | 0.004 | 0.004 | 0.050 | | 0.039 | 0.017 | 0.033 | | 0.022 | 0.019 |
| category\_fzdinen | 0.17\*\*\* | 0.073 | 0.124\* | 0.037 | 0.012 | 0.012 | 0.202\*\*\* | | 0.094 | 0.152\*\* | 0.028 | | -0.004 | 0.009 |
| category\_fzpizza | 0.000 | 0.036 | 0.027 | -0.037\*\* | -0.031\* | -0.031\* | -0.025 | | 0.012 | 0.004 | -0.041\*\* | | -0.034\*\* | -0.038\*\* |
| category\_hhclean | 0.082 | 0.045 | 0.045 | -0.002 | -0.016 | -0.016 | 0.106 | | 0.064 | 0.062 | 0.005 | | -0.014 | -0.016 |
| category\_hotdog | -0.108 | -0.033 | -0.102 | -0.037 | -0.038 | -0.038 | -0.126 | | -0.062 | -0.134 | -0.061\*\* | | -0.063\*\*\* | -0.069\*\* |
| category\_laundet | 0.142\*\* | 0.117\*\* | 0.146\*\* | 0.038 | 0.015 | 0.015 | 0.14\*\* | | 0.121\*\* | 0.133\*\* | 0.025 | | 0.003 | 0.009 |
| category\_margbut | 0.076 | 0.051 | 0.048 | -0.025 | -0.026 | -0.026 | 0.059 | | 0.035 | 0.033 | -0.026 | | -0.029 | -0.028 |
| category\_mayo | 0.106\* | 0.077 | 0.063 | -0.037\* | -0.049\*\* | -0.049\*\* | 0.122\* | | 0.098 | 0.100 | -0.010 | | -0.023 | -0.016 |
| category\_milk | 0.079 | 0.074 | 0.080 | -0.014 | -0.021 | -0.021 | 0.087 | | 0.093 | 0.110 | 0.000 | | -0.011 | -0.011 |
| category\_mustket | 0.11\*\* | 0.091\* | 0.084 | -0.012 | -0.030 | -0.030 | 0.103\* | | 0.081 | 0.068 | -0.006 | | -0.030 | -0.017 |
| category\_Paptowl | 0.489 | 0.331\*\*\* | 0.481 | -0.063 | -0.069 | -0.069 | 0.476 | | 0.311\*\*\* | 0.439 | -0.102 | | -0.107 | -0.116 |
| category\_peanbut | 0.006 | 0.034 | 0.027 | -0.05\* | -0.066\*\*\* | -0.066\*\* | 0.006 | | 0.039 | 0.036 | -0.033 | | -0.05\*\* | -0.045\*\* |
| category\_photo | 0.176\*\*\* | 0.135\*\* | 0.152\*\* | -0.013 | -0.024 | -0.024 | 0.186\*\* | | 0.144\*\* | 0.162\*\* | -0.004 | | -0.018 | -0.002 |
| category\_razors | -0.086 | -0.191 | -0.135 | 0.176 | 0.164 | 0.164 | -0.069 | | -0.173 | -0.100 | 0.204 | | 0.194 | 0.204 |
| category\_saltsnc | 0.118\*\* | 0.084\* | 0.118\*\* | 0.008 | 0.000 | 0.000 | 0.112\* | | 0.082 | 0.115\* | 0.005 | | -0.003 | -0.001 |
| category\_shamp | 0.054 | 0.021 | 0.018 | -0.039 | -0.049\*\* | -0.049\* | 0.080 | | 0.027 | 0.042 | -0.037 | | -0.052 | -0.048 |
| category\_soup | 0.148\*\*\* | 0.115\*\* | 0.095\* | 0.006 | -0.004 | -0.004 | 0.169\*\*\* | | 0.124\*\* | 0.101\* | 0.005 | | -0.006 | 0.001 |
| category\_spagsau | 0.135\*\*\* | 0.083\* | 0.095\* | 0.055\*\* | 0.024 | 0.024 | 0.167\*\*\* | | 0.112\* | 0.125\*\* | 0.062\*\* | | 0.031 | 0.042 |
| category\_sugarsu | 0.087 | 0.059 | 0.071 | -0.018 | -0.024 | -0.024 | 0.107 | | 0.073 | 0.086 | -0.018 | | -0.023 | -0.020 |
| category\_toitisu | 0.22\*\*\* | 0.198\*\*\* | 0.218\*\*\* | -0.031 | -0.043\* | -0.043 | 0.204\*\* | | 0.195\*\*\* | 0.194\*\* | -0.040 | | -0.045\*\* | -0.051\* |
| category\_toothbr | -0.009 | -0.010 | -0.005 | -0.039\* | -0.032 | -0.032 | 0.011 | | 0.006 | 0.004 | -0.039\*\* | | -0.029 | -0.031 |
| category\_toothpa | 0.080 | 0.067 | 0.067 | 0.013 | 0.001 | 0.001 | 0.119 | | 0.111 | 0.121 | -0.001 | | -0.004 | -0.003 |

1. **Conclusions, limitations and future research**

Grocery retailers have been struggling with producing accurate sales forecasts to effectively manage their inventory planning and customer satisfaction. In practice, many retailers use simple univariate models with adjustments for incoming promotional events. Some recent studies focused on taking advantage of the impact of promotional activities. For example, Gur Ali et al. (2009) proposed models with sophisticated function forms (e.g., the regression tree model) with the price and promotional information of the focal product. Huang et al. (2014) incorporated the competitive promotional information within the same product category by resorting to variable selection methods and the principle component analysis which mitigated the problem of high dimensionality. Ma et al. (2016) integrated the promotional information both within the same product category and across difficult product categories.

These conventional forecasting models all presume invariant effectiveness of marketing activities such as price reductions and feature and display promotions which may actually change over time because of the impact of many influencing factors including the change of economic condition, the change of the consumer taste, and new competition entry etc. However, these factors may not be observable or at least difficult to measure. As a result, the conventional models will be subject to structural break and potentially generate biased and less accurate forecasts.

In this study, we propose the ADL-intra-EWC model and the ADL-intra-IC model which take into account the potential forecast bias caused by the structural break. The ADL-intra-EWC model generate forecasts which are the combine of various sets of forecasts by the ADL-intra model with different estimation windows under the condition when structural breaks are detected. The ADL-intra-EWC model tries to obtain an effective trade-off between the forecast bias and the forecast error variance. In our experiment, the ADL-intra-EWC model generate the most accurate forecasts across all 30 product categories for various scenarios (e.g., forecast horizons and error measures). Table 7 shows the percentage of reductions by the model compared to other models for all the scenarios. For example, the ADL-intra-EWC model reduces the MAPE of the ADL-intra model by 0.20%, and reduces the MAPE of the Base-lift model by 6.04% based on 12 week forecast horizon. The ADL-intra-IC model tries to offset the forecast bias by adding the estimate of the forecast bias back to the error term at a cost of inflated forecast error variance given that structural breaks are detected. The ADL-intra-IC model also has superior overall forecasting performance across all the product categories, though its advantages are getting marginal for longer forecast horizons (e.g., when *h*=12, as indicated in Table 7). At the category level, our proposed models have superior forecasting performance for most of the product categories.

In this study, we also propose the ADL-own-EWC model and the ADL-own-IC model. These models are especially valuable for manufacturers when competitive promotional information cannot be accessed ([Ali and Boylan 2011](#_ENREF_2)). Under such circumstance, manufacturers need to make the best use of the data they have and produce forecasts as accurate as possible. In our experiment, the ADL-own -EWC model outperforms the ADL-own model across all the product categories for various scenarios, while the ADL-own -IC model outperforms the ADL-own model across all product category for short forecast horizons. Table 7 shows that the ADL-own-EWC model reduces the MAPE of the ADL-own model by 0.20% based on the 12 week ahead forecast horizon. At the category level, again the results are in consistent with results across all product categories.

Table 7. Forecasting performance regarding percentage reductions in various error measures

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Forecast horizon | Proposed model | Benchmark | percentage of increase/decrease | | | |
| MAPE | SMAPE | MASE | AvgRelMAE |
| h=12 | ADL-intra-EWC | ADL-intra | -0.20% | -0.48% | -0.69% | -0.70% |
| ADL-intra-IC | ADL-intra | 0.40% | 0.51% | 0.55% | 0.33% |
| ADL-own-EWC | ADL-own | -0.20% | -0.52% | -0.65% | -0.68% |
| ADL-own-IC | ADL-own | 0.51% | 0.56% | 0.56% | 0.31% |
| ADL-intra | ADL-own | -1.46% | -0.78% | -0.61% | -0.75% |
| ADL-intra-EWC | Base-lift | -6.04% | -14.52% | -11.76% | -13.98% |
| ADL-intra-IC | Base-lift | -5.48% | -13.66% | -10.66% | -13.09% |
| h=4 | ADL-intra-EWC | ADL-intra | -0.28% | -0.53% | -0.61% | -0.74% |
| ADL-intra-IC | ADL-intra | -0.66% | -0.15% | -0.41% | -0.41% |
| ADL-own-EWC | ADL-own | -0.30% | -0.55% | -0.72% | -0.75% |
| ADL-own-IC | ADL-own | -0.89% | -0.16% | -0.76% | -0.45% |
| ADL-intra | ADL-own | -1.29% | -0.88% | -1.08% | -0.93% |
| ADL-intra-EWC | Base-lift | -2.58% | -13.00% | -10.03% | -10.14% |
| ADL-intra-IC | Base-lift | -2.95% | -12.67% | -9.85% | -9.85% |
| h=1 | ADL-intra-EWC | ADL-intra | -0.83% | -0.53% | -0.51% | -1.38% |
| ADL-intra-IC | ADL-intra | -2.11% | -1.02% | -1.01% | -2.98% |
| ADL-own-EWC | ADL-own | -0.53% | -0.52% | -0.56% | -0.46% |
| ADL-own-IC | ADL-own | -2.54% | -1.12% | -1.53% | -2.93% |
| ADL-intra | ADL-own | -0.96% | -0.78% | -1.43% | -0.26% |
| ADL-intra-EWC | Base-lift | -1.44% | -11.44% | -9.99% | -2.25% |
| ADL-intra-IC | Base-lift | -2.70% | -11.88% | -10.44% | -3.83% |

There are potentials to further improve the forecasting accuracy which we leave to future research. 1) In this study, we indiscriminately restrict the smallest estimation window for the EWC method to contains 120 weeks of observations for each SKU and we combine the various sets of forecasts based on different estimation windows using equal weights. The forecasting performance may potentially be improved by choosing different smallest estimation window for different SKU’s (because their ADL models contain different number of parameters) and different forecasting combination schemes (e.g., based on k-fold evaluation). For the IC method, Clements and Hendry (1999) is a summary of different correction schemes which may have different effect on the balance of the bias and the error variance[[13]](#footnote-14). 2) Ma et al. (2016) proposed models which integrate both the intra and the inter category promotional information. We may investigate if we can further improve the forecasting performance of the ADL-intra-EWC model and the ADL-intra-IC model with inter-category information. 3) One of the alternative methods is to directly incorporate the changing process of the effectiveness of the marketing activities into the model. When the influencing factors are not observed, the changing process may be captured by an autoregressive process of the marketing activity parameters. For example, [Foekens et al. (1999)](#_ENREF_28) modelled the effectiveness (i.e., the parameters) of the price variables using the level of previous prices and the recency and the frequency of previous promotional events. The models were used to describe and evaluate the changing process of the effectiveness of the price and promotions, and their forecasting capacity was unknown. However, one of the challenges for this type of model is that it would easily go over-sophisticated when engaged with a second stage model for the parameters and therefore lose parsimony. 4) Another method alternative to the EWC method and the IC method is the impulse saturation method introduced by [Hendry and Krolzig (2001)](#_ENREF_32) and [Castle, Doornik et al. (2008)](#_ENREF_11). They proposed to saturate the ADL model with dummy variables for each of the observations and then recursively simplify the model with the *Autometrics* algorithm based on a general-to-specific modelling strategy[[14]](#footnote-15). The final model usually retains a large number of dummy variables to prevent the structural break and the subsequent forecast bias. However, this method inevitably loses information by retaining these dummy variables, which makes the forecasting performance of the method an empirical question. We leave all these above to the next stage of our research.

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1. Strictly speaking, the forecast bias comes from the change of the deterministic mean of the model due to the change of the model parameters. However, there is a very rare possibility that the deterministic mean could retain unchanged even if the parameters all change (but in a very specific way). Under such a circumstance, there will be no forecast bias even when the model is subject to structural break. However, in this study we do not consider this situation because it only happens theoretically when very restrictive conditions are met. Details based on an example of a VAR model can be found in Clements and Hendry (1999). Thereafter in this study, we assume structural breaks lead to forecast bias for the models. [↑](#footnote-ref-1)
2. This example demonstrates the issue of structural break using a simple static model. The analytical evidence for dynamic models can be found in Clements and Hendry (1999) and Pesaran and Timmerman (2005, 2007). [↑](#footnote-ref-2)
3. This setting is very common in the retailer context. In this example we artificially make up the data series but we keep the data series to be stationary. [↑](#footnote-ref-3)
4. We may have an alternative example where the sales generally decrease but also become more responsive to temporary price reductions. This may be caused by new product introduction, more competitive promotional activities by other products, or the change of economic conditions and consumer taste which are unobservable. [↑](#footnote-ref-4)
5. We use this simple example to illustrate the impact of the structural break. For this example, a model with an additional time trend variable would have better forecasting performance than the simple regression model with the full estimation window. The model with the time trend variable would still be outperformed by the model with the EWC method and the IC method. The corresponding forecasting performance is 1.129 for the MAE, 13.47% for the MAPE, and 12.84%for the SMAPE. [↑](#footnote-ref-5)
6. The Chow test is a variant of F-test which compares the fitting of the model before and after the structural break. It assumes the locations of the structural known a priori. For example, we conduct the Chow test for a specific week (e.g., week 30). The test results have a p-value which is close to 0, which indicate the null hypothesis of no structural at week 30 should be rejected. We conduct the Chow test based on most of the observations as long as there are enough observations before and after that specific week so that the test can be implemented. [↑](#footnote-ref-6)
7. To mitigate the multiple comparison problem, we may adopt very small threshold (e.g., 0.0001) for the p-value of the sequential test. [↑](#footnote-ref-7)
8. All estimates and analyses in this paper based on Information Resources, Inc. data are by the author and not by Information Resources, Inc. [↑](#footnote-ref-8)
9. We include the following US calendar events including *Halloween*, *Thanksgiving*, *Christmas*, *New Year’s Day*, *President’s Day*, *Easter*, *Memorial Day*, the *4th of July*, and *Labour Day*. [↑](#footnote-ref-9)
10. A small p-value (i.e. 0.001) is used for the sequential Chow test to mitigate the multiple testing problem [↑](#footnote-ref-10)
11. Note that, in this study, although our econometric models are based on log sales, we calculate all the error measures after we transform them back to original levels. [↑](#footnote-ref-11)
12. The values of the error measures we report in Table 4a, 4b, and 4c are all aggregate values. The values of the error measures for all forecast period may not necessarily drop in the range between the values for the non-promoted forecast period and the values for the promoted forecast period. This is because that the lengths of the promoted time periods are different across the SKU’s. As a result, the weights we use to combine the various data series for all forecast period, for the promoted forecast period, and for the non-promoted forecast period are all be different. [↑](#footnote-ref-12)
13. For example, in this study we generate the forecasts first and then add the estimated bias to all the forecasts. One of the alternative schemes is to first make adjustments to the one-step-ahead forecast, and then calculate the two-step-ahead forecast based on the value of the one-step-ahead forecast which has already adjusted, and so forth. [↑](#footnote-ref-14)
14. The algorithm recursively simplifies the model block by block pretending there are only a section of the dummy variable existing. The algorithm eventually combines all the retained dummy variables from every recursion. [↑](#footnote-ref-15)